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Flows governed by a quasi-incompressible Cahn-Hilliard-Navier-Stokes type model

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November 26, 2013
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Motivation

- $\{$CHNS type models$\} \subset \{$diffuse-interface models$\}$
- physical systems of two/**three** immiscible fluids
- cooperation with *Glass Service*
    - float glass process (glass/tin/nitrogen system)
    - **numerical simulations using a thermodynamically constistent three-component model** (derived by O. Souček)
- example of numerical results

Motivation

- {CHNS-Fourier type models} $\subset$ {diffuse-interface models}
- physical systems of two/**three** immiscible fluids (including thermal effects)
- cooperation with *Glass Service*
  - float glass process (glass/tin/nitrogen system)
  - **numerical simulations using a thermodynamically constistent three-component model** (derived by O. Souček)
- example of numerical results

Motivation

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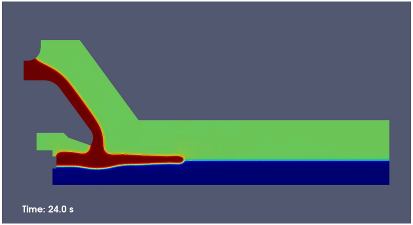
- cooperation with *Glass Service*
  - float glass process (glass/tin/nitrogen system)
  - **numerical simulations using a thermodynamically constistent three-component model** (derived by O. Souček)
- example of numerical results



Boyer, F. and C. Lapuerta (2006). Study of a three component Cahn-Hilliard flow model. *ESAIM: Mathematical Modelling and Numerical Analysis 40*, 653–687.

Preliminary research

Two-component quasi-incompressible CHNS model

$$\varrho \partial_t c + \varrho \mathbf{v} \cdot \nabla c = \operatorname{div}(\beta \nabla \mu),$$

$$\mu = \frac{1}{\varrho}\left(24\sigma\varepsilon^{-1}F'(c) - \frac{3}{2}\sigma\varepsilon\Delta c\right) - r_\star p,$$

$$\operatorname{div} \mathbf{v} = -r_\star \operatorname{div}(\beta \nabla \mu),$$

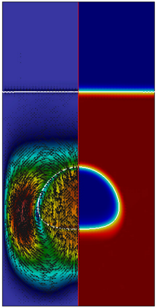
$$\varrho \partial_t \mathbf{v} + \varrho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla\left(p + \frac{2}{3}\nu \operatorname{div} \mathbf{v}\right) + \operatorname{div}(2\nu\mathbb{D}) + \varrho(\mu + r_\star p)\nabla c + \varrho \mathbf{b}.$$

- variant of a model derived in Lowengrub and Truskinovsky (1998)
- the density ϱ is an explicit function of c
- $F(c) = \frac{1}{2}c^2(1-c)^2$ ... double-well potential (free energy)
- $r_\star = 0$ for matching densities ("Model H")

Lowengrub, J. and L. Truskinovsky (1998). Quasi-incompressible Cahn-Hilliard fluids and topological transitions. *R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci. 454*(1978), 2617–2654.

Numerical simulations

- energy-stable numerical schemes
- implementation in FEniCS (`http://fenicsproject.org`)
- quantification of the differences between the proposed model and the existing class of competing models
- testing the code on various benchmark problems
- simulations with realistic physical parameters



Aland, S. and A. Voigt (2011). Benchmark computations of diffuse interface models for two-dimensional bubble dynamics. *Int. J. Numer. Meth. Fluids 69*(3), 747–761.