Nečas Center for Mathematical Modeling

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ERC-CZ project LL1202 - MORE

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Stability of Flows of Incompressible Stress Power-law Fluids

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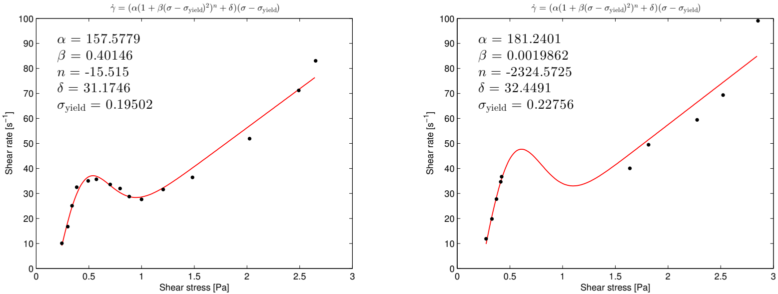
Stress Power-law fluids

$$\mathbb{D} = \alpha \left(1 + \beta \left|\mathbb{T}_\delta\right|^2\right)^n \mathbb{T}_\delta$$



(a) Behavior under constant applied shear stress.

(b) Behavior under constant applied shear rate.

Figure: Steady-state stress/shear-rate behavior for a 7.5/7.5 mM TTAA/NaSal solution.

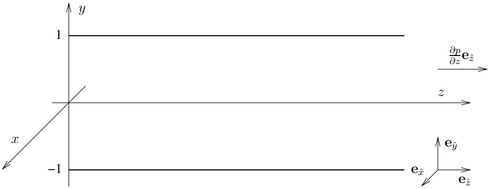


Figure: Plane Poiseuille flow.

Linearized equations for disturbances

$$\rho\left(\frac{\partial \mathbf{v}'}{\partial t} + [\nabla \bar{\mathbf{v}}]\,\mathbf{v}' + \left[\nabla \mathbf{v}'\right]\bar{\mathbf{v}}\right) = -\nabla p' + \operatorname{div} \mathbb{T}'_\delta$$

$$\operatorname{div} \mathbf{v}' = 0$$

$$\mathbb{D}' = \alpha\left(1 + \beta\left|\bar{\mathbb{T}}_\delta\right|^2\right)^n \left(\mathbb{T}'_\delta + \frac{2n\beta}{1+\beta|\bar{\mathbb{T}}_\delta|^2}(\bar{\mathbb{T}}_\delta : \mathbb{T}'_\delta)\bar{\mathbb{T}}_\delta\right)$$

$$\mathbf{v}'\Big|_{y=\pm 1} = 0$$

Linearized Stability

- **Wavelike solution**

$$\mathbf{v}'(y,z,t) = \tilde{\mathbf{v}}'(y)\mathrm{e}^{i\lambda z - i\omega t}$$
$$\mathbb{T}'_\delta(y,z,t) = \tilde{\mathbb{T}}'_\delta(y)\mathrm{e}^{i\lambda z - i\omega t}$$

$\lambda\ldots$ *streamwise wave number*, $\omega\ldots$ *frequency*

- **Generalized Eigenvalue problem**

$$\mathbb{A}\mathbf{x} = \omega\mathbb{B}\mathbf{x}$$

- Solved using the **pseudospectrall collocation method**
- Flow is **stable** if

$$\mathrm{imag}(\omega) < 0$$

- *Note:* for $n = 0$ or $\beta = 0$, the problem reduces to the classical **Orr-Sommerfeld equation**

See you in the Marble Hall!