Poster: Computer Analysis and Simulation of Strain Limiting Models With Linearized Strain

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Strain limitting model - motivation

Materials with strain limitting behavior (norm of strain is bounded):

- Soft tissues
- Polymers
- Brittle materials

Nomenclature

- $\boldsymbol{T}$ ... Chauchy stress
- $\boldsymbol{u}$ ... Displacement vector
- $\boldsymbol{E}$ ... Green-St. Venant tensor $\boldsymbol{E} = \frac{1}{2}(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T + (\nabla \boldsymbol{u})^T \nabla \boldsymbol{u})$
- ε ... Linearized strain $\varepsilon = \frac{1}{2}(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)$

Implicit constitutive theory

- $\hat{\boldsymbol{G}}(\boldsymbol{E}, \boldsymbol{T}) = \mathbf{0}$, special explicit case $\boldsymbol{E} = \boldsymbol{G}(\boldsymbol{T})$

Strain limitting model

Proposed in [Rajagopal, 2010]

$$\boldsymbol{E} = \gamma\left(1 - \exp\frac{-\lambda \operatorname{tr}\boldsymbol{T}}{1+\|\boldsymbol{T}\|}\right)\boldsymbol{I} + \frac{\boldsymbol{T}}{2\mu\left(1+\kappa\|\boldsymbol{T}\|^{a}\right)^{\frac{1}{a}}}. \qquad (1)$$

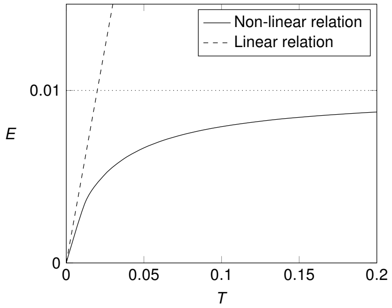


Figure: One dimensional stress strain relation for $\mu = 1$, $a = 0.9$ and $\kappa = 24.8$.

Linearizing stress tensor

Small displacement gradient assumption

- Assuming $\|\nabla \boldsymbol{u}\|_\infty \ll 1$
- Under this assumption we can approximate Green-St. Venant strain tensor $\boldsymbol{E}$ by linearized strain ε
- Neglecting term $((\nabla \boldsymbol{u})^T \nabla \boldsymbol{u})/2$

- $\|\nabla \boldsymbol{u}\|_\infty \ll 1$ is not always true for Hookean model applications (Linear Elastic Fracture Mechanics). Stress limitting model may fullfill the condition $\|\nabla \boldsymbol{u}\|_\infty \ll 1$ by proper setting of model parameters.
- Linearizing $\boldsymbol{E}$ does not mean to linearize constitutive relation.
- Strain limitting model is actively studied from the point of view of modeling, analysis and numerics.

System of equations for tensor $\boldsymbol{T}$, tensor ε and vector $\boldsymbol{u}$

$$\operatorname{div} \boldsymbol{T} = \boldsymbol{f}, \qquad \boldsymbol{T} = \boldsymbol{T}^T, \tag{2a}$$

$$\varepsilon = \boldsymbol{G}(\boldsymbol{T}), \tag{2b}$$

$$\varepsilon = \frac{1}{2}\left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right). \tag{2c}$$

System of equations for tensor $\boldsymbol{T}$ and tensor ε

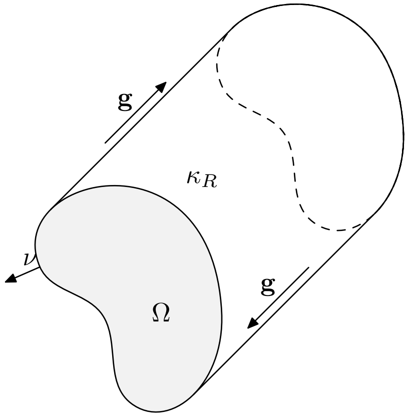
$$\operatorname{div} \boldsymbol{T} = \boldsymbol{f}, \tag{3a}$$

$$\varepsilon = \boldsymbol{G}(\boldsymbol{T}), \tag{3b}$$

$$\varepsilon_{ij,kl} + \varepsilon_{kl,ij} - \varepsilon_{ik,jl} - \varepsilon_{jl,ik} = 0, \qquad i,j,k,l \in 1,2,3. \tag{3c}$$

Antiplane stress

In antiplane stress problem the only nonzero components of the stress tensor $\boldsymbol{T}$ are T_{13} and T_{23}. $T_{11} = T_{22} = T_{33} = T_{12} = 0$. We also assume that T_{23} and T_{13} depend only on x_1 and x_2.



V shaped cutout, variable α loaded (top and bottom) by shear force F.

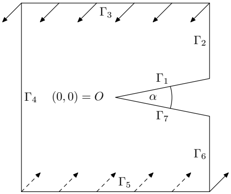
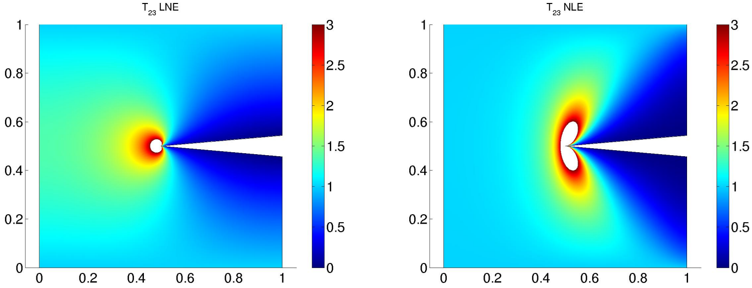


Figure: Computational domain.

Aim is to compare linear elasticity model with nonlinear strain limiting model both with linearized strain tensor ε. For details see [Kulvait et al., 2013].

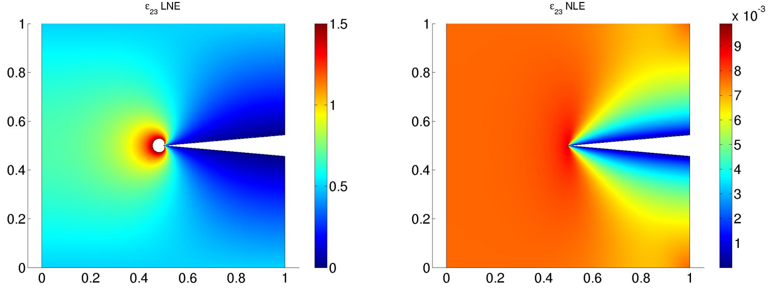
Stress distribution comparison

Figure: Comparison of T_{23} for linear (left) and nonlinear (right) model.



Strain distribution comparison

Figure: Comparison of ε_{23} linear (left) and nonlinear (right) model.



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