Nečas Center for Mathematical Modeling

MOdelling REvisited + MOdel REduction

ERC-CZ project LL1202 - MORE

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Blood flow modelling

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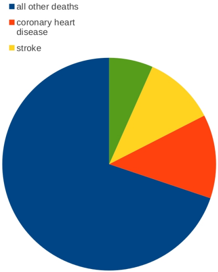
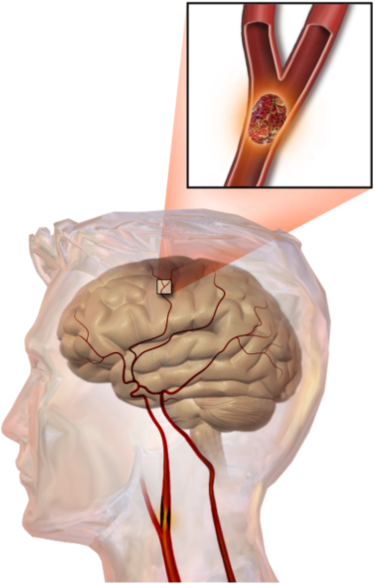
A little of statistics



- in 2008 died 57million people
- from that number 17.3 million people died of cardivascular diseases
  - 7.3 million died of coronary heart disease
  - 6.2 million died of stroke

Source:Global status report on noncommunicable disaeses 2010. Geneva, World Health Organization, 2011.

Stroke



The need for blood flow modelling

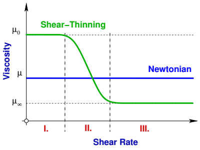
- there is a wide range of cardiovascular diseases
- many of them are related to blood coagulation
- such diseases can be avoided by medication (anticoagulants)

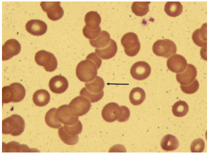
☞ it is necessary to understand blood coagulation process as accurately as possible

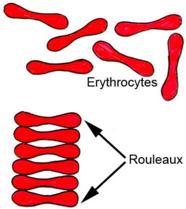
- in several studies was shown that blood coagulation process must be described both from biochemical and rheological point of view

☞ the model must couple the rheological equations together with biochemical reaction equations

- blood exhibits shear-thinning behaviour (creation of rouleaux)







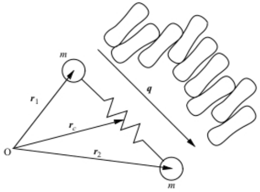
- elasticity (the membranes of red blood cells, erythrocytes, are elastic)

Demands on the blood model

- we know from experiments that red blood cells tend to populate the middle of the vessel
- red blood cells push chemical species and blood platelets towards the vessel walls

☞ proper model of blood model should predict the higher concentrations of chemical species in the vicinity of vessel walls

Remarks concerning the derivation of the model



- Moyers-Gonzalez, M., Owens, R.G. , Fang, J.F. (2008) A non-homogeneous constitutive model for human blood. Part 1. Model derivation and steady flow
- R. G. Owens, A new microstructure-based constitutive model for human blood

Equations to be solved

$$Re\frac{D\mathbf{u}}{Dt} - 2\eta_s \nabla \cdot \dot{\boldsymbol{\gamma}} - \nabla \cdot \boldsymbol{\tau} + \nabla p = 0 \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\frac{D\hat{N}}{Dt} + \frac{1}{2}b(\dot{\gamma})(\hat{N} - \hat{N}_{st})(\hat{N} + \hat{N}_{st} - 1) = 0 \tag{3}$$

$$\boldsymbol{\tau} + De(\frac{\partial \boldsymbol{\tau}}{\partial t} + (\mathbf{u} \cdot \nabla)\boldsymbol{\tau} - \nabla \mathbf{u} \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot \nabla \mathbf{u}^T) = 2De\dot{\boldsymbol{\gamma}}(\mathbf{u}) \tag{4}$$

where $b(\dot{\gamma})$ corresponds to fragmentation rate, $\hat{N}_{st}$ is the value of $\hat{N}$ at steady simple shear flow with shear rate $\dot{\gamma}$, De Deborah number, which is function of shear rate $\dot{\gamma}$ and average rouleaux size $\hat{N}$

Numerical treatment

- FEM library deal.ii
- computations of Jacobians using automatic differentiation (package of TRILINOS Sacado)
- DEVSS
- SUPG