

Fluid Model of Crystal Plasticity

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Motivations

- ▶ ultra-fine grain formation by SPD

General Aim

- ▶ derive a thermodynamically admissible model in Eulerian coordinates
- ▶ suitable for numerical simulations
- ▶ analytical properties

Computational Results

- ▶ 2-turn ECAE
- ▶ FE solver of fully coupled problem

Do not hesitate to stop over during the poster session.

Fluid Model of Crystal Plasticity

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Introduction

Constitutive stress-plate deformation experiments are a motivation, the plastic behavior of crystalline solids is treated as a flow of highly viscous, incompressible material. Treating them classical single crystal hypothesis we present a purely Eulerian set of equations describing flow of a plastic material. Numerical simulations for a 2-turn equal channel angular extrusion are reported.

Motivations and Aims

Basic plastic deformation
 - large deformation of sheet during deformation
 - large strain (simple compression with multiple rotations)
 - large change of shape and cross section
 - change of internal properties (grain refinement) (SPC)

Goals:
 - derivation of a fluid model of crystal plasticity (classical)
 - a general constitutive thermodynamically admissible
 - solution for formation of ultra-fine grains
 - find a steady state microstructure
 - suitable for interaction with an experimental data.

Kinematics

Kinematic decomposition $F = F_p F_e$
 F_p the plastic deformation, stretch and rotation of the lattice
 F_e the elastic deformation of the lattice due to formation of dislocations
 $\dot{F} = \dot{F}_p + \dot{F}_e$
 $\dot{F}_p = \dot{F} - \dot{F}_e = \dot{F} - \dot{F}_e F_e^{-1} F_e$

The Cauchy Stress T

The Balance Equations
 $\text{Div } T = \rho_0 \dot{v}$
 $T = \rho_0 \dot{v} - \text{TW} - \text{TW}^T + \rho_0 \dot{C} D_t$
 where ρ_0 is the density, \dot{v} is the velocity, $\dot{C} D_t$ is the Cauchy stress rate.

Crystal Plasticity Assumption

By using \dot{F}_p as governed by power law
 $\dot{F}_p = \text{sym}(\dot{F}_p) + \text{skw}(\dot{F}_p)$
 where $\text{sym}(\dot{F}_p)$ is the symmetric part and $\text{skw}(\dot{F}_p)$ is the skew-symmetric part.
 The second law of thermodynamics is satisfied
 $\dot{C} : T - \text{Div } \Theta_p = \sum_{\alpha} \dot{\gamma}_{\alpha} (\dot{\gamma}_{\alpha} + \dot{\gamma}_{\alpha}^2) - \sum_{\alpha} \dot{\gamma}_{\alpha} \left(\frac{\dot{\gamma}_{\alpha}}{\dot{\gamma}_{\alpha}} \right) \geq 0$

Use Evolution of Slip Systems

The system are strongly coupled with plastic variables and rotation
 $\dot{\gamma}_{\alpha} = \dot{\gamma}_{\alpha}^0 + \dot{\gamma}_{\alpha}^1 + \dot{\gamma}_{\alpha}^2 + \dots$
 We introduce internal variables in form of rotation angles θ_{α} (e.g. $\theta_{\alpha} = \theta_{\alpha}^0 + \theta_{\alpha}^1 + \theta_{\alpha}^2$)
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$

Global Model

The unknowns are velocity v , Cauchy stress T , density ρ and the direction \hat{d}^{α}
 $\dot{\rho} + \text{Div } (\rho v) = 0$
 $\text{Div } T = \rho_0 \dot{v}$
 $T = \rho_0 \dot{v} - \text{TW} - \text{TW}^T + \rho_0 \dot{C} D_t$
 $\dot{\gamma}_{\alpha} = \text{sym}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
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2-Turn Equal Channel Angular Extrusion/Numerical Treatment

The flow direction is along the x_1 axis and the working surface is $x_2 = 0$.
 - Material properties: Young's Modulus $E = 210 \text{ GPa}$, Poisson's ratio $\nu = 0.33$ and work hardening parameter $n = 0.02$ (see [2]).
 - Initial conditions: $\theta_{\alpha} = 0$, $\dot{\gamma}_{\alpha} = 0$, $\dot{\theta}_{\alpha} = 0$, $\rho = 1$, $v = 0$, $T = 0$.
 - Boundary conditions: $\dot{\gamma}_{\alpha} = 0$, $\dot{\theta}_{\alpha} = 0$, $\rho = 1$, $v = 0$, $T = 0$.
 - Meshing: $100 \times 100 \times 100$ (see [2]).
 - Numerical solution: $\dot{\gamma}_{\alpha} = 0$, $\dot{\theta}_{\alpha} = 0$, $\rho = 1$, $v = 0$, $T = 0$.
 - For more details see the paper [2].

Discrete Evolution

Finite element approximation
 $\dot{\gamma}_{\alpha} = \text{sym}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$

We use the following approximation
 $\dot{\gamma}_{\alpha} = \text{sym}(\dot{F}_p)_{\alpha}$
 $\dot{\theta}_{\alpha} = \dot{\gamma}_{\alpha} \text{skw}(\dot{F}_p)_{\alpha}$
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The Results of Numerical Simulations

Solutions were calculated on a small domain of 2D mesh (2000 nodes) in software package (FEM).

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