Nečas Center for Mathematical Modeling

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MINISTRY OF EDUCATION, YOUTH AND SPORTS

Qualitative analysis for a generalization of the Darcy-Forchheimer equations

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Equations we study

Let $\Omega \subset \boldsymbol{R}^d$ be a Lipschitz domain and $\partial\Omega = \Gamma_1 \cup \Gamma_2$.

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We look for $(\boldsymbol{m}, \boldsymbol{v}, p) : \Omega \to \boldsymbol{R}^d \times \boldsymbol{R}^d \times \boldsymbol{R}$ solving

$$
\begin{aligned}
\nabla p + \boldsymbol{m} &= \boldsymbol{f} \quad \text{in } \Omega, \\
\boldsymbol{v} &= 0 \quad \text{in } \Omega, \\
\boldsymbol{h}(\boldsymbol{m}, \boldsymbol{v}, p) &= \boldsymbol{0} \quad \text{in } \Omega, \\
(\boldsymbol{v} - \boldsymbol{v}_0) \cdot \boldsymbol{n} &= 0 \quad \text{on } \Gamma_1, \\
p - p_0 &= 0 \quad \text{on } \Gamma_2.
\end{aligned}
$$

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where $\mathcal{A}$ denotes a *maximal monotone r-graph*, i.e. $\mathcal{A} \subset \boldsymbol{R}^d \times \boldsymbol{R}^d \times \boldsymbol{R}$ satisfies each of the following:

(A1) *inclusion of the origin*

(A2) *monotonicity*

(A3) *maximality*

(A4) r*-coercivity*

(A5) there exists a *Carathéodory selection* $\boldsymbol{m}^* : \boldsymbol{R}^d \times \boldsymbol{R} \to \boldsymbol{R}^d$

Results

Theorem 1. (Principal existence theorem)

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Theorem 2. (Maximum and minimum principle)

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Theorem 3. (Extended existence theorem)

Conclusion

Proved existence of solutions for situations like Barus' law, i.e.

$$\nabla p + \mu_1 \exp(\mu_2 p)\boldsymbol{v} = \boldsymbol{f}.$$

Bidding farewell

Thank you for your attention

&

see you by the pictureless poster.