Mixed Least Squares Finite Element Methods for Hyperelastic Materials

Introduction to the poster

Benjamin Müller

MORE workshop 2013 in Liblice
November 24 - 27, 2013

Faculty of Mathematics
University of Duisburg - Essen

joint work with Jörg Schröder, Alexander Schwarz and Karl Steeger
Gerhard Starke

supported by the German research foundation (DFG)

November 26th, 2013

Motivation

- FEM is an important tool for the simulation of **elasticity** and plasticity in solid mechanics:
  - linear vs. **nonlinear** model
  - different discretization methods are possible (Galerkin, Mixed FEM, **LSFEM**, . . .)

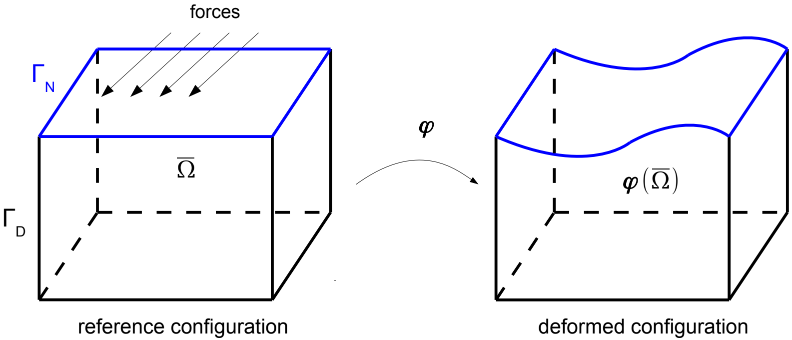
Motivation

- FEM is an important tool for the simulation of **elasticity** and plasticity in solid mechanics:
  - linear vs. **nonlinear** model
  - different discretization methods are possible (Galerkin, Mixed FEM, **LSFEM**, ...)
- Aims of our research:
  - investigate **suitability of least squares finite element methods** in this context and develop/improve them
  - numerical scheme should work in the case of (quasi-) **incompressible** materials
  - approximate displacements **and** stresses → **mixed formulation**

Motivation

- FEM is an important tool for the simulation of **elasticity** and plasticity in solid mechanics:
  - linear vs. **nonlinear** model
  - different discretization methods are possible (Galerkin, Mixed FEM, **LSFEM**, . . .)
- Aims of our research:
  - investigate **suitability of least squares finite element methods** in this context and develop/improve them
  - numerical scheme should work in the case of (quasi-) **incompressible** materials
  - approximate displacements **and** stresses → **mixed formulation**
- advantages of LSFEM:
  - provides candidate for a-posteriori error estimation → **adaptive refinement**
  - combinations of FEM spaces not restricted (inf-sup condition)

Problem description for given body $\Omega \subset \mathbb{R}^3$



- body force: given by a density $\mathbf{f} : \Omega \to \mathbb{R}^3$
- surface force: given by a density $\mathbf{g} : \Gamma_N \to \mathbb{R}^3$
- deformation: $\boldsymbol{\varphi} : \bar{\Omega} \to \mathbb{R}^3$
- $\boldsymbol{\varphi} = \mathbf{id} + \mathbf{u}$ with the pointwise displacement $\mathbf{u} : \bar{\Omega} \to \mathbb{R}^3$
- first Piola - Kirchhoff stress tensor: $\mathbf{P} : \bar{\Omega} \to \mathbb{R}^{3\times 3}$

Least Squares Finite Element approach

- homogeneous isotropic hyperelastic material of Neo-Hookean type with

$$\psi(\mathbf{C}) = \frac{\mu}{2}\operatorname{tr}(\mathbf{C}) + \frac{\lambda}{4}\det\mathbf{C} - \left(\frac{\lambda}{2}+\mu\right)\ln\sqrt{\det\mathbf{C}}$$

- Minimization of the nonlinear least squares functional

$$\mathcal{F}(\mathbf{P},\mathbf{u}) = \omega_1^2\|\operatorname{div}\mathbf{P}+\mathbf{f}\|^2 + \omega_2^2\|\mathcal{A}(\mathbf{P}\mathbf{F}(\mathbf{u})^T) - \mathbf{B}(\mathbf{u})\|^2$$

  in appropriate spaces.
- Algorithmic implementation:
  - Gauss-Newton scheme for minimization of $\mathcal{F}(\mathbf{P},\mathbf{u})$
  - Raviart-Thomas elements $\mathcal{RT}_1$ for $\mathbf{P}$ and piecewise quadratic continuous elements for $\mathbf{u}$
  - backtracking line search as damping strategy

Numerical examples and conclusion

- Numerical examples (for the incompressible case):
  1. detection of critical loads (2d plane strain)
  2. adaptivity (2d Cook's membrane, plane strain)
  3. 3d Cook's membrane with plot of the normal components of the approximated stress tensor

- Conclusion:
  - satisfactory approximations of $\mathbf{P}$ and $\mathbf{u}$
  - approach works in the incompressible case
  - least squares functional tends to identify the regions of interest

Thank you for your attention