Heat conduction problem of an evaporating liquid wedge

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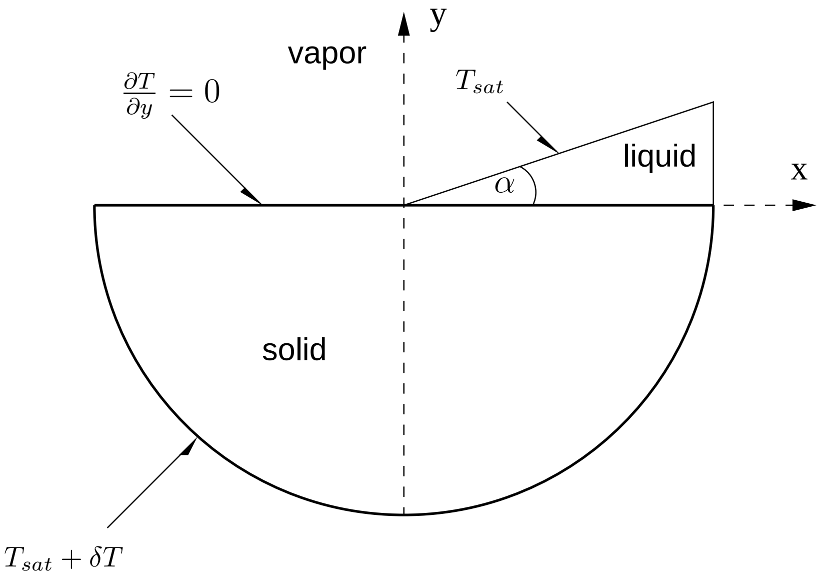
Problem



y
vapor
$\frac{\partial T}{\partial y} = 0$
T_{sat}
liquid
α
x
solid
$T_{sat} + \delta T$

Equations

$$\Delta T = 0, \qquad \text{in } M \tag{0}$$

$$T = T_{\text{sat}} + \delta T, \qquad \text{on } \Gamma_1 \tag{1}$$

$$\frac{\partial T}{\partial y} = 0, \qquad \text{on } \Gamma_2 \tag{2}$$

$$k_S \frac{\partial T}{\partial y} = k_L \frac{T_{\text{sat}} - T}{\alpha x}, \qquad \text{on } \Gamma_3 \tag{3}$$

Functional setting

- $T \in H^1(M)$ ... the unknown temperature of the solid
- $\gamma_{iD}(T) \in H^{1/2}(\Gamma_i)$ ... Dirichlet trace for Γ_i
- $\gamma_{iN}(T) \in H^{-1/2}(\Gamma_i)$ ... Neumann trace for Γ_i
- $L^2_{xdx}(0,1) \ldots \int_0^1 |v(x)|^2 x dx < \infty$
- $L^2_{dx/x}(0,1) \ldots \int_0^1 |v(x)|^2 \frac{dx}{x} < \infty$

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Main theorem

$$
\begin{aligned}
\Delta T &= 0, && \text{in } M && (0)\\
T &= \theta, && \text{on } \Gamma_1 && (1)\\
\frac{\partial T}{\partial Y} &= 0, && \text{on } \Gamma_2 && (2)\\
K\frac{\partial T}{\partial Y} &= -\frac{T+\sigma}{X}, && \text{on } \Gamma_3 && (3)
\end{aligned}
$$

Theorem.
$\forall \theta \in H^{1/2}(\Gamma_1)$, $\sigma \in H^{1/2}(\Gamma_3)$
$\exists!$ solution $T \in H^1(M)$ such that
$\gamma_{3N}(T) \in H^{-1/2} \cap L^2_{xdx}(\Gamma_3)$, $\gamma_{3D}(T) + \sigma \in L^2_{dx/x}$

Ideas of the proof

- new unknown function:
  $T \in H^1(M) \iff \quad \tau \in H^{-1/2}(\Gamma_3)$
- (3) $\iff$ $(\mathrm{I} - \mathcal{B})\tau = f$
- Fredholm Theorem

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Further problems

- $\int_0^1 \tau$, $\int_0^x \tau = ??$
- singularity/regularity in 0
- numerical solution
- weaker functional setting
- more general model

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