Modeling and simulation of float glass forming process

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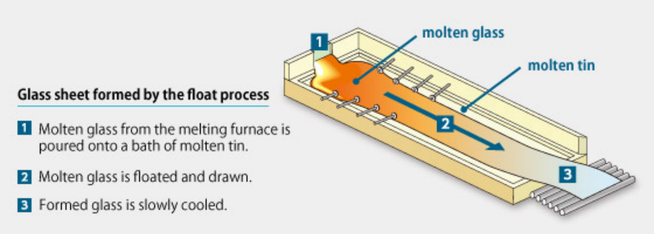
Pilkington process

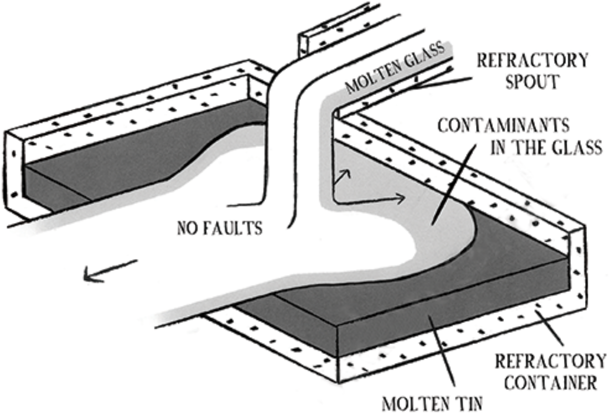
The process for manufacturing of flat glass. Invented by Alastair Pilkington in 1952.



Source: Nippon Electric Glass Co., Ltd.

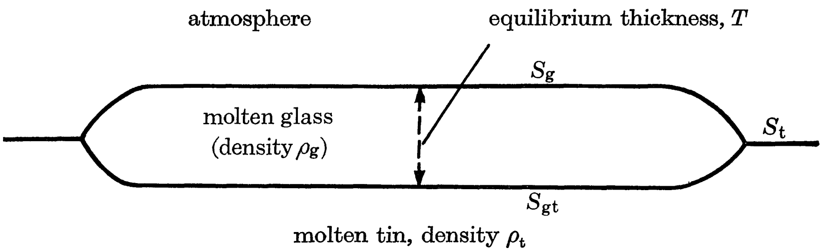
L. A. B. Pilkington. Review lecture. The float glass process.
Proc. R. Soc. A-Math. Phys. Eng. Sci., 314(1516):1–25, 1969

Pilkington process – glass contamination



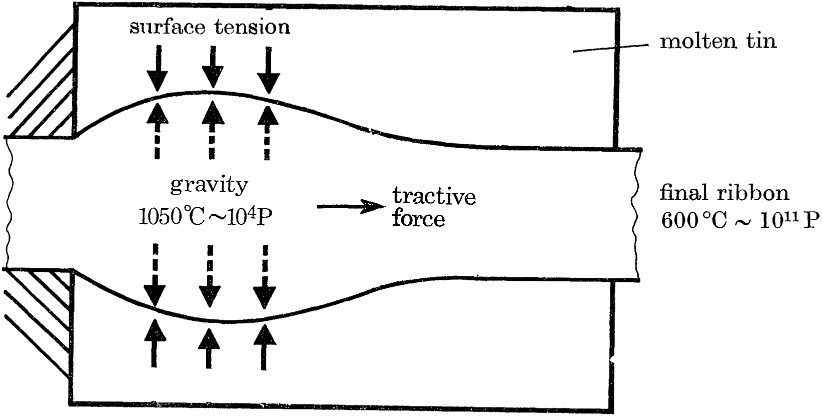
L. A. B. Pilkington. Review lecture. The float glass process.
Proc. R. Soc. A-Math. Phys. Eng. Sci., 314(1516):1–25, 1969

Pilkington process – equilibrium thickness



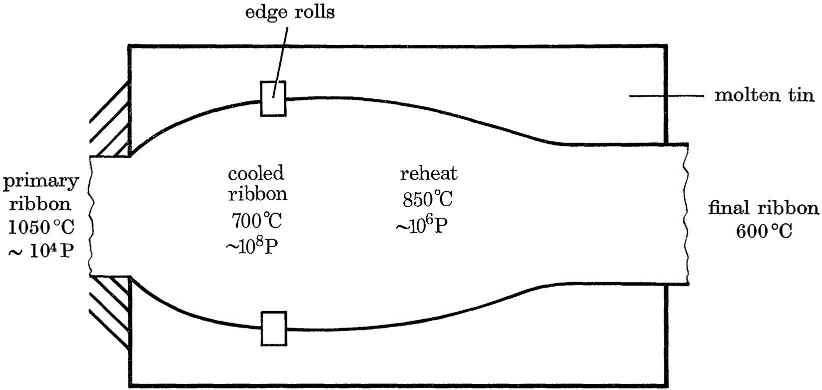
L. A. B. Pilkington. Review lecture. The float glass process. *Proc. R. Soc. A-Math. Phys. Eng. Sci.*, 314(1516):1–25, 1969

Pilkington process – top view, forces acting on the ribbon



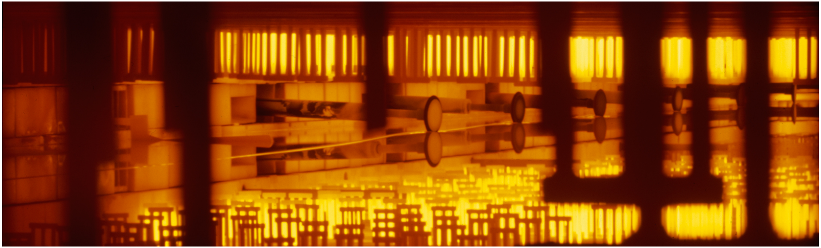
L. A. B. Pilkington. Review lecture. The float glass process.
Proc. R. Soc. A-Math. Phys. Eng. Sci., 314(1516):1–25, 1969

Pilkington process – top view, temperature/viscosity variations



L. A. B. Pilkington. Review lecture. The float glass process. *Proc. R. Soc. A-Math. Phys. Eng. Sci.*, 314(1516):1–25, 1969

Pilkington process – reality



Source: Corning Museum of Glass

Challenges and design goals

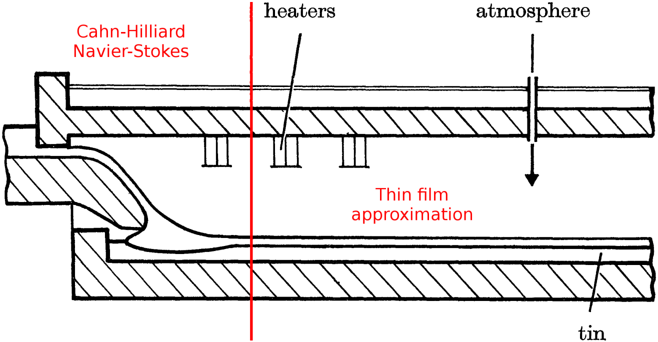
Challenges:

- Multicomponent system with free boundaries.
- Surface tension. (Surface tension effects on walls.)
- Large temperature/viscosity variations.
- Multiple length scales.

Design goals:

- Thickness control. (Edge rolls placement, temperature distribution, *lehr* speed.)
- Minimal contamination. (Spout dimensions.)

- Tin bath entrance: Navier–Stokes–Cahn–Hilliard model.
- Tin bath (ribbon): Thin film approximation.



Three components Cahn–Hilliard–Navier–Stokes model

Three-component Cahn–Hilliard–Navier–Stokes model

$$\frac{\partial c_i}{\partial t} + \mathbf{v} \cdot \nabla c_i = \operatorname{div}\left(\frac{M_0(\mathbf{c})}{\Sigma_i} \nabla \mu_i\right), \quad \forall i \in \{1,2,3\},$$

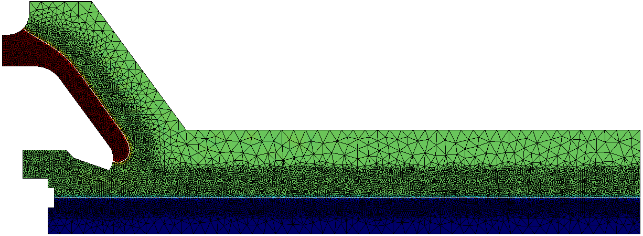
$$\mu_i = \frac{4\Sigma_T}{\varepsilon} \sum_{j \neq i} \left(\frac{1}{\Sigma_j} \left(\partial_i F(\mathbf{c}) - \partial_j F(\mathbf{c})\right)\right) - \frac{3}{4} \varepsilon \Sigma_i \Delta c_i, \quad \forall i \in \{1,2,3\},$$

$$\operatorname{div} \mathbf{v} = 0,$$

$$\varrho \left(\frac{\partial}{\partial t} \mathbf{v} + (\mathbf{v} \cdot \nabla)\, \mathbf{v}\right) + \nabla p - \operatorname{div}(2\nu \mathbf{D}) = \sum_{j=1}^{3} \mu_j \nabla c_j + \varrho \mathbf{g}.$$

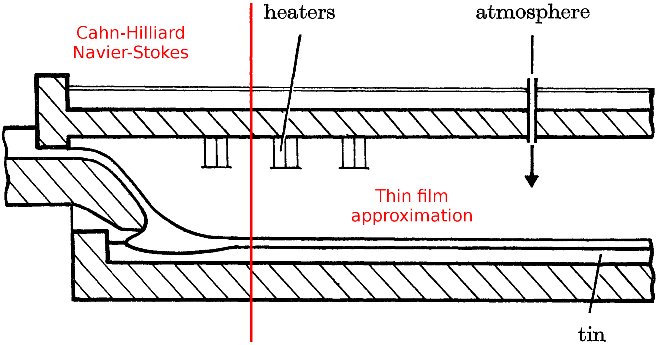
Franck Boyer and Céline Lapuerta. Study of a three component Cahn-Hilliard flow model. *ESAIM: Mathematical Modelling and Numerical Analysis*, 40:653–687, 7 2006

Cahn–Hilliard–Navier–Stokes – results



Pilkington process – complementary models

- Tin bath entrance: Navier–Stokes–Cahn–Hilliard model.
- Tin bath (ribbon): Thin film approximation.



Thin film approximation

Obtained by averaging Navier–Stokes equations with respect to vertical variable.

Thin film approximation

$$\frac{\partial H}{\partial t} + \mathrm{div}(H\mathbf{v}) = 0$$

$$\rho_{\mathrm{g}} \left[\frac{\partial}{\partial t} (H\mathbf{v}) + \mathrm{div}\left(H\mathbf{v} \otimes \mathbf{v} \right) \right] = \mathrm{div}\, \mathbb{T}_{\mathrm{gs}}$$

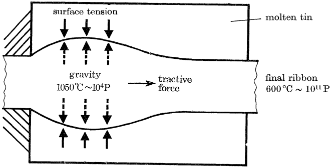
$$P =_{\mathrm{def}} \rho_{\mathrm{g}} \left(1 - \frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{t}}} \right) g \frac{H^2}{2}$$

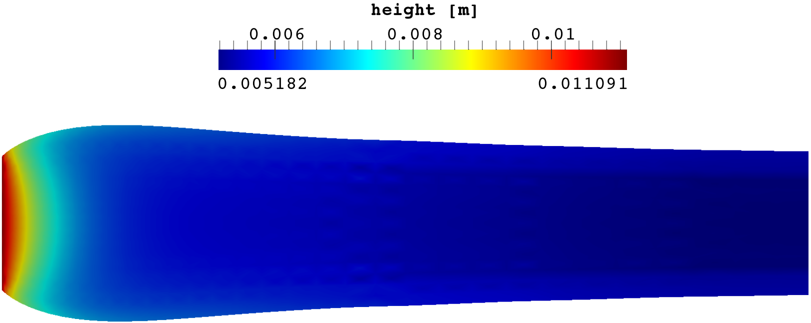
$$\mathbb{T}_{\mathrm{gs}} =_{\mathrm{def}} -P\mathbb{I} + 2\mu H \left(\mathrm{div}\, \mathbf{v} \right) \mathbb{I} + 2\mu H \mathbb{D}$$

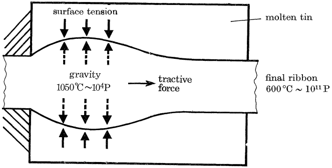
V.V. Popov. Flow of a viscous film over the surface of an inviscid fluid.
Journal of Applied Mechanics and Technical Physics, 23:188–194, 1982

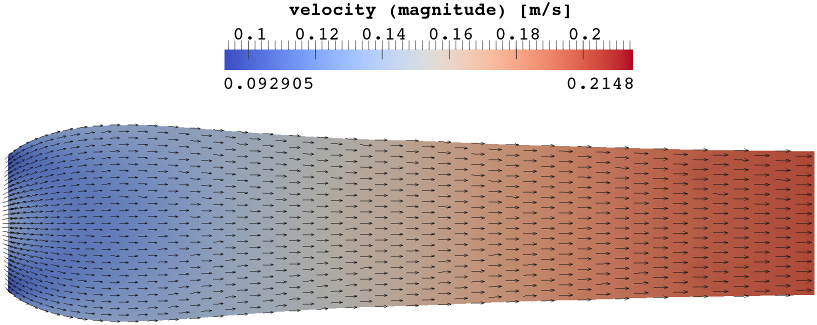
V. V. Popov. Flow of a viscous film with free boundaries in the one-dimensional approximation.
Journal of Applied Mechanics and Technical Physics, 24:507–509, 1983

Thin film approximation – results

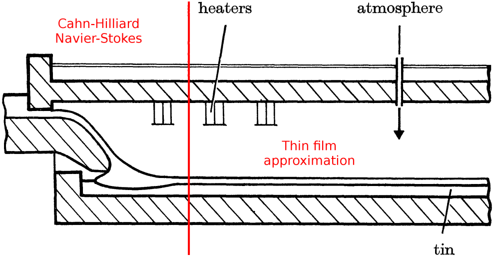





surface tension
molten tin
gravity
1050℃~10⁴P
tractive
force
final ribbon
600 °C ~ 10¹¹ P


velocity (magnitude) [m/s]
0.1 0.12 0.14 0.16 0.18 0.2
0.092905
0.2148

Conclusion



- Cahn–Hilliard–Navier–Stokes type model for the initial phase of the process.
- Reduced model (thin film approximation) for the terminal phase of the process.
- Coupling of both models. (Plus the model for tin flow.)

Implementation

General purpose library for solution of differential equations using finite element method.

http://fenicsproject.org

Acknowledgement

Support of Glass Service (an advanced solutions supplier in the field of glass melting, conditioning and forming) is gratefully acknowledged.

Thank you for your attention.