Why is there a need for implicit constitutive relations?

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- Interestingly, if one considers the classical ideal gas law or Boyles law and Charles law, they used either stress (or a component of it) in terms of the specific volume, temperature, etc., or the specific volume in terms of the stress, etc. They did not care much with regard to which form they used, not because they clearly appreciated philosophical or fundamental issues with regard to modeling.
- Later Classical Models like the Euler fluid model, the Navier-Stokes fluid model, the Cauchy elastic model, the linearized elastic model, etc., provide explicit expression for the Cauchy stress in terms of other variable.
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Philosophical Imperative

Newton is unequivocal about the fact that force is the cause and motion is the effect as evidenced by the following sentiments:

> *"The causes by which true and relative motion are distinguished, one from the other, are the forces impressed upon bodies to generate motion."*
> *"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*
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"About two years since I printed this Theory as an anagram at the end of my Book of the Descriptions of Helioscopes, viz ceiiinosssttuu, idest, Ut tension sic vis; That is, the Power of any Spring is in the same proportion of the tension thereof: that is if one Power stretch or bend it one space, two will bend it two, three will bend it three and so forward.

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"The classical theory of elasticity formulates the intuitive notion of spring in a basic hypothesis: For a given body, there exists a fixed natural state, generally regarded as unstressed, such that the body when constrained into a form near to that in its natural state and then released from all external forces returns perfectly to this natural state independently of the manner in which the forces have been applied and removed."

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"Force has a unique position among all possible basic concepts in physical science since it may be regarded as having a direct relation to the concept of cause. Indeed, many students of the problem, and foremost among these the Kantian school of thought, consider force the exact physical formulation of cause and causality. According to this point of view, natural science relates all phenomena in nature to certain substrata, the phenomena being considered as their effects."

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A constitutive equation is a relation between forces and motions. In popular terms, force is applied to a body to cause" it to undergo a motion, and the motion caused" differs according to the nature of the body. In continuum mechanics the forces of interest are contact forces, which are specified by the stress tensor $\mathbf{T}$ *.*

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David Hume believed that the connection between cause and effect is a necessary assumption if one is to conduct scientific enquiry. He is not convinced that one can necessarily state with certainty a specific cause with regard to a specific effect. However, he feels that there seems to be a clear connection between that which is called cause and that which is referred to as the effect, and that there is a power in one of them, the cause for producing the effect. The following remarks makes his position clear on this issue: *"When we look about us towards external objects, and consider the operation of causes, we are never able, in a single instance, to discover any power or necessary connexion- - -.We only find, that the one does actually, in fact, follow the other." "But when one particular species of event has always, in all instances, been conjoined with another, we make no longer any scruple of foretelling one upon the appearance of the other...We then call that one object, Cause; the other, Effect. We suppose that there is some connexion between them; some power in the one, by which it infallibly produces the other, and operates with the greatest certainty and strongest necessity."*

Articulates well the need for implicit relations.

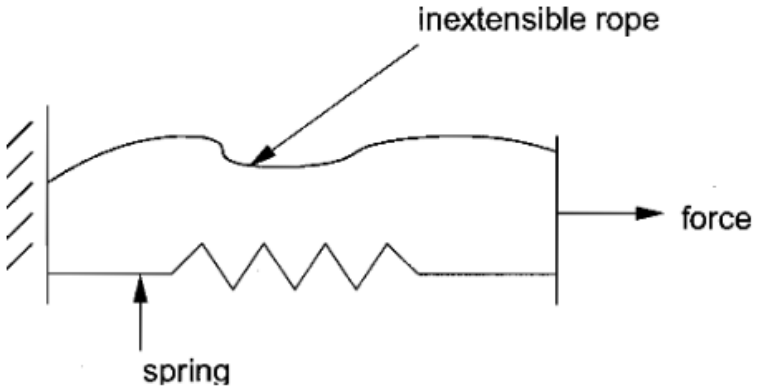

Figure : A linear spring and an inextensible rope in parallel. The spring need not be a linear spring

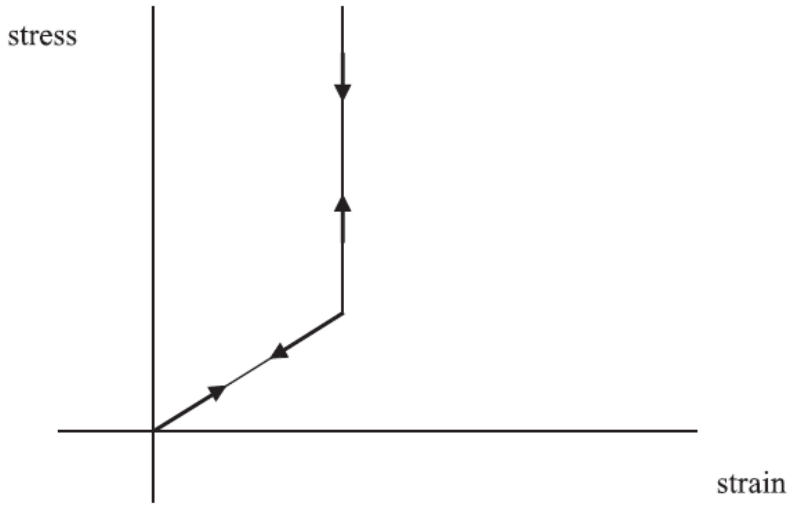

Figure : Non-dissipative response wherein the stress cannot be expressed explicitly as a function

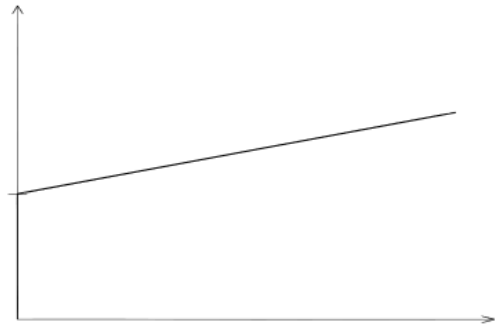

Figure : The response of a Bingham like mass-dashpot system.

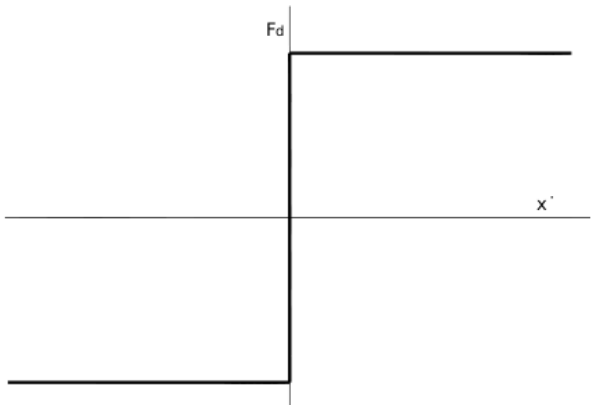

Figure : Coulomb Friction

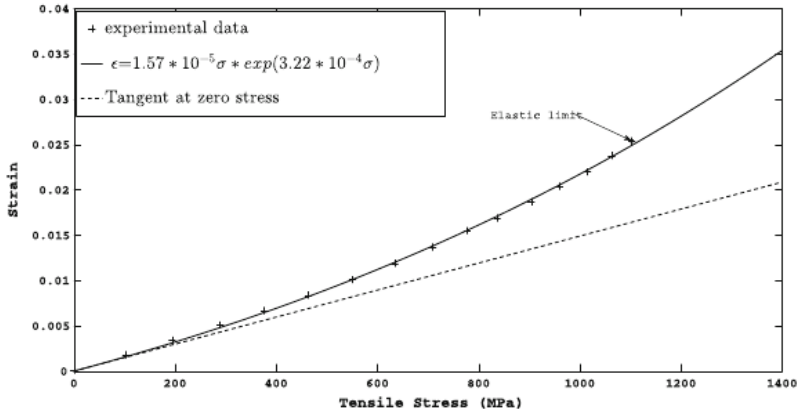

Figure : Non-linear relationship between the strain and stress in the small strain range for a Gum metal alloy (Figure is taken from the paper by Saito et al.

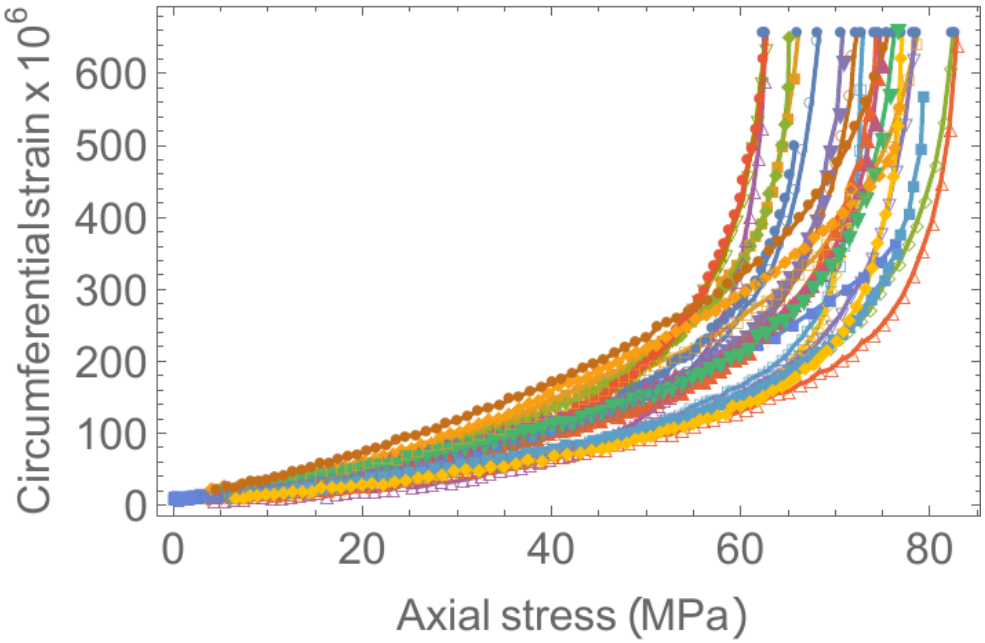

Figure : circumferential strain versus the axial stress

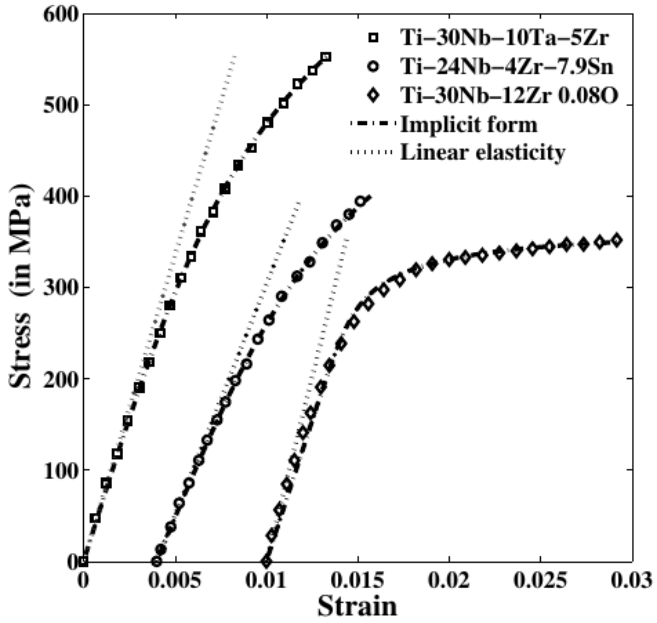

Figure : Uniaxial data sourced from Sakaguch et al. (2004), Hao et al. (2005) and Hou et al. (2010)

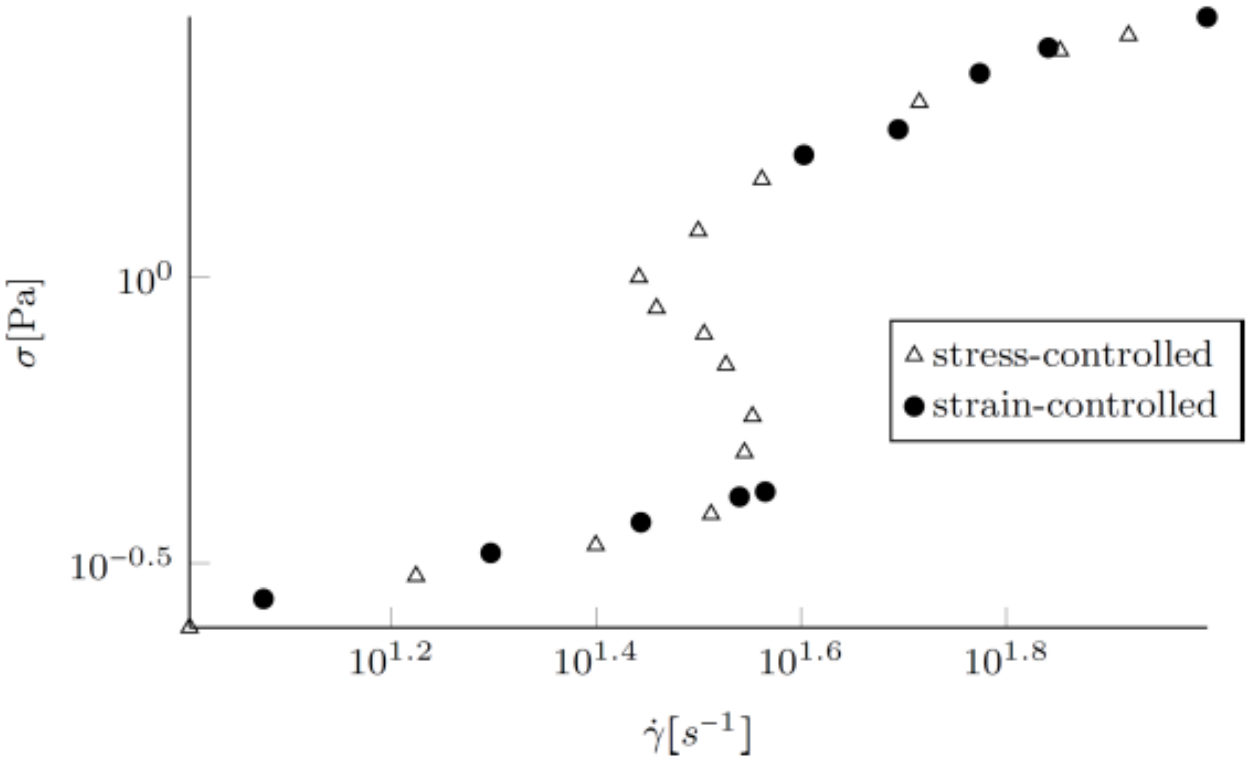

Figure : Shear stress versus shear rate in the experiments by Boltenhagen et al. (1997) for Tris (2-hydroxyethyl) tallowalkyl ammonium acetate (TTAA) surfactant dissolved in water containing sodium salicylate (NaSal).

- Linearization of Cauchy elasticity or Green elasticity cannot describe the response elastic responses that are displayed.
- Fluid models wherein the stress is related to the history of kinematics cannot describe the data response for the colloidal fluid that is shown.
- Implicit constitutive relations can describe all the responses that have been depicted.
- Such implicit models are necessary to describe materials that undergo structural changes with time and deformation.

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- There are several important problems wherein classical theories lead to absurd and physically meaningless results.
- One very important example is Fracture Mechanics, the development of cracks and their movements.
- Already it has been shown conclusively that one can describe fractures within the context of implicit theories.
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- Much of what has been done with implicit constitutive theories concerns homogeneous bodies. One needs to look at implicit constitutive relations for inhomogeneous bodies.

- The most important open problem in physics, that of turbulence, has not been adequately described as it is incorrectly assumed that it is the fluctuation in the velocity and the pressure (just a part of the stress) lead to turbulence.

- Implicit theories are ideally suited to describe damage, fatigue and failure of bodies.
- But that as Scheherazade would say "Is another story".