Modelling the Flow of Traffic

K. R. Rajagopal

Texas A&M University
College Station, Texas 77845

"These people have lost the power of astonishment at their own action. When they give birth to a fantastic action or a foolish law they do not stop or stare at the monster that they have brought forth.

– G. K. Chesterton (The Mad Official).

"These people have lost the power of astonishment at their own action. When they give birth to a fantastic action or a foolish law they do not stop or stare at the monster that they have brought forth.

– G. K. Chesterton (The Mad Official).

"We must always remember that mathematics deals with abstract models and that different models can describe the same empirical situation with various degrees of approximation and simplicity. The manner in which mathematical theories are applied does not depend on preconceived ideas and is not a matter of logic; it is a purposeful technique which depends on, and changes with, experience.

– W. Feller

"We must always remember that mathematics deals with abstract models and that different models can describe the same empirical situation with various degrees of approximation and simplicity. The manner in which mathematical theories are applied does not depend on preconceived ideas and is not a matter of logic; it is a purposeful technique which depends on, and changes with, experience.

– W. Feller

Outline of this lecture

- Need for modelling the flow of traffic.
- Current approaches.
- Drawbacks of current approaches.
- Simplified Lumped Parameter Discrete Dynamical System Model
- Full Discrete Dynamical System Model

Outline of this lecture

- Need for modelling the flow of traffic.
- Current approaches.
- Drawbacks of current approaches.
- Simplified Lumped Parameter Discrete Dynamical System Model
- Full Discrete Dynamical System Model

Outline of this lecture

- Need for modelling the flow of traffic.
- Current approaches.
- Drawbacks of current approaches.
- Simplified Lumped Parameter Discrete Dynamical System Model
- Full Discrete Dynamical System Model

Outline of this lecture

- Need for modelling the flow of traffic.
- Current approaches.
- Drawbacks of current approaches.
- Simplified Lumped Parameter Discrete Dynamical System Model
- Full Discrete Dynamical System Model

Outline of this lecture

- Need for modelling the flow of traffic.
- Current approaches.
- Drawbacks of current approaches.
- Simplified Lumped Parameter Discrete Dynamical System Model
- Full Discrete Dynamical System Model

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Difficulties in developing a theory for the flow of traffic

- Human-in-the-loop.
- Need to take into account human factors like the psychology of the individual drivers.
- Need to take into consideration weather conditions.
- Need to take into accound road conditions.
- Need to take into account the condition of the vehicle.
- Need to take into account heterogeneity of traffic.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Attribute of a model/theory

- Ability to explain a large class of observed phenomena
- Simplicity
- Aesthetics
- Economy of thought
- Predictive Capability
- Consilience of Induction
- Falsifiability, etc.

Popular approaches to modelling the flow of traffic

- Macroscopic Modelling
  - Continuum models (Eulerian)- Lighthill-Whitham-Richards (LWR) model and their generalizations, Models based on Kinetic Theory
  - Other sundry models
- Microscopic Modelling
  - Car-following models (Lagrangian)

Popular approaches to modelling the flow of traffic

- Macroscopic Modelling
  - Continuum models (Eulerian)- Lighthill-Whitham-Richards (LWR) model and their generalizations, Models based on Kinetic Theory
  - Other sundry models
- Microscopic Modelling
  - Car-following models (Lagrangian)

Popular approaches to modelling the flow of traffic

- Macroscopic Modelling
  - Continuum models (Eulerian)- Lighthill-Whitham-Richards (LWR) model and their generalizations, Models based on Kinetic Theory
  - Other sundry models
- Microscopic Modelling
  - Car-following models (Lagrangian)

Popular approaches to modelling the flow of traffic

- Macroscopic Modelling
  - Continuum models (Eulerian)- Lighthill-Whitham-Richards (LWR) model and their generalizations, Models based on Kinetic Theory
  - Other sundry models
- Microscopic Modelling
  - Car-following models (Lagrangian)

Popular approaches to modelling the flow of traffic

- Macroscopic Modelling
  - Continuum models (Eulerian)- Lighthill-Whitham-Richards (LWR) model and their generalizations, Models based on Kinetic Theory
  - Other sundry models
- Microscopic Modelling
  - Car-following models (Lagrangian)

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Continuum Models - Summary

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Continuum Models - Summary

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Continuum Models - Summary

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Continuum Models - Summary

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Continuum Models - Summary

- Early model due to Lighthill and Whitham, and Richards (LWR model). Subsequent generalizations include those due to Payne, Papageorgiou and his coworkers.
- Treat traffic as continuum.
- Governing equations based on two physical principles
  - Balance of mass.
  - Balance of linear momentum.
- In the LWR model inertia of the traffic is neglected.

Drawbacks of continuum models: ill-defined “density”:

- Quote from L. M. Milne-Thomson, Theoretical Hydrodynamics, Dover 1996: *“If* $\mathbf{M}$ *is the mass of the fluid within a closed volume* $\mathbf{V}$ *, we can write*

$$\mathbf{M} = \mathbf{V}\rho_{\mathbf{1}}, \tag{1}$$

and $\rho_{\mathbf{1}}$ *is then the average density of the fluid within the volume at that instant. In a hypothetical medium continuously distributed we can define the density* ρ *as the limit of* $\rho_{\mathbf{1}}$ *when* $\mathbf{V} \to \mathbf{0}$*.*
In an actual fluid which consists of a large number of individual molecules we cannot let $\mathbf{V} \to \mathbf{0}$*, for at some stage there might be no molecules within the volume* $\mathbf{V}$*.*

We must therefore be content with a definition of density given by (1) *on the understanding that the dimensions of* $\mathbf{V}$ *are to be made very small, but not so small that* $\mathbf{V}$ *does not still contain a large number of molecules. In air at ordinary temperatures there are about* $\mathbf{3 \times 10^{19}}$ *molecules per* $\mathbf{cm^3}$*. A sphere of radius* $\mathbf{0.001cm}$*. will then contain about* $\mathbf{10^{11}}$ *molecules and although small in the hydrodynamical sense will be reasonably large for the purposes of measuring average density."*

> *"Suppose that you could mark the molecules in a glass of water; then pour the contents of the glass into the ocean and stir the latter thoroughly so as to distribute the marked molecules uniformly throughout the seven seas; if then you took a glass of water anywhere out of the ocean, you would find in it about hundred of your marked molecules."*
>
> *– Lord Kelvin (Quote from E. Schrodinger's book)*

- Upshot: Density as in fluids is ill-defined as a mathematical term for the flow of traffic

> *"Suppose that you could mark the molecules in a glass of water; then pour the contents of the glass into the ocean and stir the latter thoroughly so as to distribute the marked molecules uniformly throughout the seven seas; if then you took a glass of water anywhere out of the ocean, you would find in it about hundred of your marked molecules."*
>
> *– Lord Kelvin (Quote from E. Schrodinger's book)*

- Upshot: Density as in fluids is ill-defined as a mathematical term for the flow of traffic

"Suppose that you could mark the molecules in a glass of water; then pour the contents of the glass into the ocean and stir the latter thoroughly so as to distribute the marked molecules uniformly throughout the seven seas; if then you took a glass of water anywhere out of the ocean, you would find in it about hundred of your marked molecules."

– Lord Kelvin (Quote from E. Schrodinger's book)

- Upshot: Density as in fluids is ill-defined as a mathematical term for the flow of traffic

- Constitutive relationship between the speed and density of the traffic. (Fundamental Traffic Characteristic).
- In the generalizations of LWR model due to Payne and others
  - Inertia of traffic is not neglected
  - Traffic is treated as a Navier-Stokes fluid.

- Constitutive relationship between the speed and density of the traffic. (Fundamental Traffic Characteristic).
- In the generalizations of LWR model due to Payne and others
  - Inertia of traffic is not neglected
  - Traffic is treated as a Navier-Stokes fluid.

- Constitutive relationship between the speed and density of the traffic. (Fundamental Traffic Characteristic).
- In the generalizations of LWR model due to Payne and others
  - Inertia of traffic is not neglected
  - Traffic is treated as a Navier-Stokes fluid.

- Constitutive relationship between the speed and density of the traffic. (Fundamental Traffic Characteristic).
- In the generalizations of LWR model due to Payne and others
    - Inertia of traffic is not neglected
    - Traffic is treated as a Navier-Stokes fluid.

Models based on the Kinetic Theory of gases

Summary: Originally proposed by Newell and subsequent contributions due to Prigogene and Andrews, and others

- Treat traffic as an assemblage of molecules and appeal to kinetic theory.
- Momentum transfer is akin to collision between molecules. Collision is used to denote interaction between two cars on the highway to slow the car behind.
- Passing is the main relaxation mechanism as the density of traffic in the other lanes is assumed to be small.

Summary: Originally proposed by Newell and subsequent contributions due to Prigogene and Andrews, and others

- Treat traffic as an assemblage of molecules and appeal to kinetic theory.
- Momentum transfer is akin to collision between molecules. Collision is used to denote interaction between two cars on the highway to slow the car behind.
- Passing is the main relaxation mechanism as the density of traffic in the other lanes is assumed to be small.

Models based on the Kinetic Theory of gases

Summary: Originally proposed by Newell and subsequent contributions due to Prigogene and Andrews, and others

- Treat traffic as an assemblage of molecules and appeal to kinetic theory.
- Momentum transfer is akin to collision between molecules. Collision is used to denote interaction between two cars on the highway to slow the car behind.
- Passing is the main relaxation mechanism as the density of traffic in the other lanes is assumed to be small.

"Consider in the first instance a "simple gas, that is, a gas composed of molecules which are all alike. Let the mass of any molecule be $\mathbf{m}$*. Let* $\mathbf{dr}$ *denote a small volume surrounding the point* $\mathbf{r}$*, which is large enough to contain a great number of molecules, while still possessing dimensions small compared with the scale of variation of such macroscopic quantities as the pressure, temperature, or mass-velocity of the gas. (For example, a cube of edge one-hundredth of a millimetre contains about* $\mathbf{2.687 \times 10^{10}}$ *molecules in a gas "at standard temperature and pressure. . . . ") "*

– S. Chapman and T. G. Cowling (Quote from, The mathematical theory of non-uniform gases: An account of the kinetic theory of viscosity, thermal conduction and diffusion in gases, Cambridge University Press, third edition, 1990.)

"Consider in the first instance a "simple gas, that is, a gas composed of molecules which are all alike. Let the mass of any molecule be $\mathbf{m}$*. Let* $\mathbf{dr}$ *denote a small volume surrounding the point* $\mathbf{r}$*, which is large enough to contain a great number of molecules, while still possessing dimensions small compared with the scale of variation of such macroscopic quantities as the pressure, temperature, or mass-velocity of the gas. (For example, a cube of edge one-hundredth of a millimetre contains about* $\mathbf{2.687 \times 10^{10}}$ *molecules in a gas "at standard temperature and pressure. . . . ") "*

– S. Chapman and T. G. Cowling (Quote from, The mathematical theory of non-uniform gases: An account of the kinetic theory of viscosity, thermal conduction and diffusion in gases, Cambridge University Press, third edition, 1990.)

Other sundry models - Cellular Automata

- Due to Nagel and Schrackenberg.
- Space is discretized into cells and time is discretized as well.
- Cars move from one cell to the other in the forward direction.
- Number of cells moved in a unit time is proportional to the speed.

- Due to Nagel and Schrackenberg.
- Space is discretized into cells and time is discretized as well.
- Cars move from one cell to the other in the forward direction.
- Number of cells moved in a unit time is proportional to the speed.

Other sundry models - Cellular Automata

- Due to Nagel and Schrackenberg.
- Space is discretized into cells and time is discretized as well.
- Cars move from one cell to the other in the forward direction.
- Number of cells moved in a unit time is proportional to the speed.

Other sundry models - Cellular Automata

- Due to Nagel and Schrackenberg.
- Space is discretized into cells and time is discretized as well.
- Cars move from one cell to the other in the forward direction.
- Number of cells moved in a unit time is proportional to the speed.

Four rules to update the state of a vehicle:

- Increase current speed by a unit if it has not reached maximum,
- Update speed of a vehicle to be the minimum of speed in the earlier step and the number of cells to the next vehicle,
- With a probability p, reduce the speed of vehicles in the earlier step by a unit, and
- update the position accordingly.

Four rules to update the state of a vehicle:

- Increase current speed by a unit if it has not reached maximum,
- Update speed of a vehicle to be the minimum of speed in the earlier step and the number of cells to the next vehicle,
- With a probability p, reduce the speed of vehicles in the earlier step by a unit, and
- update the position accordingly.

Four rules to update the state of a vehicle:

- Increase current speed by a unit if it has not reached maximum,
- Update speed of a vehicle to be the minimum of speed in the earlier step and the number of cells to the next vehicle,
- With a probability p, reduce the speed of vehicles in the earlier step by a unit, and
- update the position accordingly.

Four rules to update the state of a vehicle:

- Increase current speed by a unit if it has not reached maximum,
- Update speed of a vehicle to be the minimum of speed in the earlier step and the number of cells to the next vehicle,
- With a probability p, reduce the speed of vehicles in the earlier step by a unit, and
- update the position accordingly.

Additional drawbacks of generalizations of LWR Models

- Spurious propagation of disturbances - bi-directional interactions in fluids - pointed out by Daganzo.
- Not clear why traffic should be modelled as a Navier-Stokes fluid
- No transparency in incorporating heterogeneity resulting from vehicles or sections with different characteristics.

Additional drawbacks of generalizations of LWR Models

- Spurious propagation of disturbances - bi-directional interactions in fluids - pointed out by Daganzo.
- Not clear why traffic should be modelled as a Navier-Stokes fluid
- No transparency in incorporating heterogeneity resulting from vehicles or sections with different characteristics.

Additional drawbacks of generalizations of LWR Models

- Spurious propagation of disturbances - bi-directional interactions in fluids - pointed out by Daganzo.
- Not clear why traffic should be modelled as a Navier-Stokes fluid
- No transparency in incorporating heterogeneity resulting from vehicles or sections with different characteristics.

Drawbacks of models based on the kinetic theory of gases

- Not enough number of vehicles on the road to get the order of numbers of collisions required.
- Time scales are of significantly different orders of magnitude - hence, treating traffic and gases in a similar way is a problem.
- Main Relaxation mechanism is passing - models are not applicable to single lane highways.
- Spurious propagation of disturbances.

Drawbacks of models based on the kinetic theory of gases

- Not enough number of vehicles on the road to get the order of numbers of collisions required.
- Time scales are of significantly different orders of magnitude - hence, treating traffic and gases in a similar way is a problem.
- Main Relaxation mechanism is passing - models are not applicable to single lane highways.
- Spurious propagation of disturbances.

Drawbacks of models based on the kinetic theory of gases

- Not enough number of vehicles on the road to get the order of numbers of collisions required.
- Time scales are of significantly different orders of magnitude - hence, treating traffic and gases in a similar way is a problem.
- Main Relaxation mechanism is passing - models are not applicable to single lane highways.
- Spurious propagation of disturbances.

Drawbacks of models based on the kinetic theory of gases

- Not enough number of vehicles on the road to get the order of numbers of collisions required.
- Time scales are of significantly different orders of magnitude - hence, treating traffic and gases in a similar way is a problem.
- Main Relaxation mechanism is passing - models are not applicable to single lane highways.
- Spurious propagation of disturbances.

A comparison of the order of variables

A comparison of the order of variables

Char. Variable		Order of Values	
Traffic	Continuum/ Kinetic Theory	Traffic	Continuum/ Kinetic Theory
Vehicle length (L_c)	Molecule dia. (d_m)	O(m)	O(nm)

A comparison of the order of variables

Char. Variable		Order of Values	
Traffic	Continuum/ Kinetic Theory	Traffic	Continuum/ Kinetic Theory
Vehicle length (L_c)	Molecule dia. (d_m)	O(m)	O(nm)
Following Distance (Δ)	Mean free path (l_m)	O(m)	O(nm)

A comparison of the order of variables

Char. Variable		Order of Values	
Traffic	Continuum/ Kinetic Theory	Traffic	Continuum/ Kinetic Theory
Vehicle length (L_c)	Molecule dia. (d_m)	O(m)	O(nm)
Following Distance (Δ)	Mean free path (l_m)	O(m)	O(nm)
Section length(L_s)	Char. Length (L)	O(m)	O(mm)
	Char. Vol. Of flow domain (V)	O(m)	O(mm^3)

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Ratio	Continuum/Kinetic theory	Ratio	Traffic
$\frac{\Delta}{L_c}$	1-6	$\frac{l_m}{d_m}$	0.1-10

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Ratio	Continuum/Kinetic theory	Ratio	Traffic
$\frac{\Delta}{L_c}$	1-6	$\frac{l_m}{d_m}$	0.1-10
$\frac{L_s}{\Delta}$	50-500	$\frac{L}{l_m}$	$O(10^6)$

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Relevant ratios for comparison

Three ratios

- Following distance/Length of section
- Length of vehicle/Length of section
- Length of vehicle/Following distance

Ratio	Continuum/Kinetic theory	Ratio	Traffic
$\frac{\Delta}{L_c}$	1-6	$\frac{l_m}{d_m}$	0.1-10
$\frac{L_s}{\Delta}$	50-500	$\frac{L}{l_m}$	$O(10^6)$
$\frac{L_s}{L_c}$	50-500	$\left(\frac{L}{d_m}\right)^3$	$O(10^{18})$

Time Scale: For traffic - $\mathbf{O(sec)}$ and for molecules $\mathbf{O(10^{-9}sec)}$.

Development of a discrete dynamical system model - An overview

- Issues to be handled
- Approach
- Corroboration with data from US 183 in Austin, Texas (by V. Tyagi) and Rajiv Gandhi Salai, Chennai (C. Shankar and Lelitha Devi).

Development of a discrete dynamical system model - An overview

- Issues to be handled
- Approach
- Corroboration with data from US 183 in Austin, Texas (by V. Tyagi) and Rajiv Gandhi Salai, Chennai (C. Shankar and Lelitha Devi).

Development of a discrete dynamical system model - An overview

- Issues to be handled
- Approach
- Corroboration with data from US 183 in Austin, Texas (by V. Tyagi) and Rajiv Gandhi Salai, Chennai (C. Shankar and Lelitha Devi).

Objective of modelling and issues

Objective: To predict the evolution of speed and the number of vehicles in a section of a road.

Issues:

- Reflect the balance of number of vehicles in a section.
- Expressible in terms of physical, and if possible, measurable variables.
- Provide ways to refine the model - account for heterogeneity of vehicles in traffic and/or devices on-board vehicles.
- Eulerian description.

Objective of modelling and issues

Objective: To predict the evolution of speed and the number of vehicles in a section of a road.

Issues:

- Reflect the balance of number of vehicles in a section.
- Expressible in terms of physical, and if possible, measurable variables.
- Provide ways to refine the model - account for heterogeneity of vehicles in traffic and/or devices on-board vehicles.
- Eulerian description.

Objective of modelling and issues

Objective: To predict the evolution of speed and the number of vehicles in a section of a road.

Issues:

- Reflect the balance of number of vehicles in a section.
- Expressible in terms of physical, and if possible, measurable variables.
- Provide ways to refine the model - account for heterogeneity of vehicles in traffic and/or devices on-board vehicles.
- Eulerian description.

Objective of modelling and issues

Objective: To predict the evolution of speed and the number of vehicles in a section of a road.

Issues:

- Reflect the balance of number of vehicles in a section.
- Expressible in terms of physical, and if possible, measurable variables.
- Provide ways to refine the model - account for heterogeneity of vehicles in traffic and/or devices on-board vehicles.
- Eulerian description.

Variables

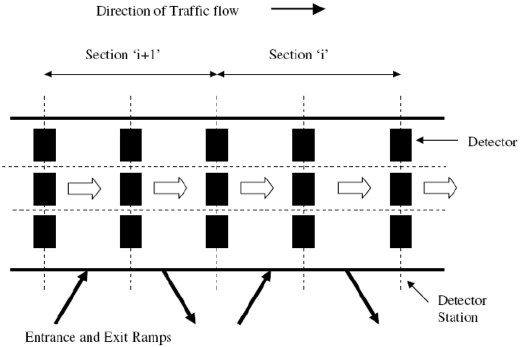

Figure : Division of a road into sections

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

Variables

- State:
  - N - number of vehicles in a section
  - $\bar{\triangle}$ - average following distance in a section, and
  - $\bar{v}$ - average speed of vehicles in a section.
- Section: For pragmatic reasons, a section has a loop detector at its ends and one in the middle. Measurements at the middle detector assumed to be representative of the state associated with the section.
- Geometric Parameters:
  - L_s - length of the section,
  - L_c - length of the car.
- Other parameters - Specific to constitutive equations for the evolution of speed.
- Subscript i refers to the i^{th} section.

A discrete approach to modelling

- Traffic is a large collection of interconnected dynamical systems (vehicles).
  - Interconnections result from safe vehicle following.
- Central Hypothesis: Existence of a few vehicles in the highway whose state is representative of the state of the traffic in the section.
  - "Average driver in a section.
  - Specific to a section - hence, conversion from Lagrangian to Eulerian description is automatically taken care of.

A discrete approach to modelling

- Traffic is a large collection of interconnected dynamical systems (vehicles).
    - Interconnections result from safe vehicle following.
- Central Hypothesis: Existence of a few vehicles in the highway whose state is representative of the state of the traffic in the section.
    - "Average driver in a section.
    - Specific to a section - hence, conversion from Lagrangian to Eulerian description is automatically taken care of.

A discrete approach to modelling

- Traffic is a large collection of interconnected dynamical systems (vehicles).
  - Interconnections result from safe vehicle following.
- Central Hypothesis: Existence of a few vehicles in the highway whose state is representative of the state of the traffic in the section.
  - "Average driver in a section.
  - Specific to a section - hence, conversion from Lagrangian to Eulerian description is automatically taken care of.

A discrete approach to modelling

- Traffic is a large collection of interconnected dynamical systems (vehicles).
  - Interconnections result from safe vehicle following.
- Central Hypothesis: Existence of a few vehicles in the highway whose state is representative of the state of the traffic in the section.
  - "Average driver in a section.
  - Specific to a section - hence, conversion from Lagrangian to Eulerian description is automatically taken care of.

A discrete approach to modelling

- Traffic is a large collection of interconnected dynamical systems (vehicles).
    - Interconnections result from safe vehicle following.
- Central Hypothesis: Existence of a few vehicles in the highway whose state is representative of the state of the traffic in the section.
    - “Average driver in a section.
    - Specific to a section - hence, conversion from Lagrangian to Eulerian description is automatically taken care of.

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

where,

1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

  where,

  1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
  2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
  3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

where,

1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

where,

1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

where,

1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Balance of number of vehicles:

$$\dot{N}_i = \dot{N}_i^{en} - \dot{N}_i^{ex} + \dot{\tilde{n}}_i \tag{2}$$

where,

1. $\dot{N}_i^{en}$:the rate of vehicles entering the given section,
2. $\dot{N}_i^{ex}$:the rate of vehicles exiting the given section
3. $\dot{\tilde{n}}_i$: the net inflow into the section from the ramps

- Compatibility: $\dot{N}_{i+1}^{ex} = \dot{N}_i^{en}$
- Constitutive equation: $\dot{N}_i^{ex} = \frac{\bar{v}_i}{\bar{\triangle}_i + L_c}$.

Governing Equations

- Constitutive equations for car-following:

$$\dot{\bar{\triangle}}_i = \frac{(\bar{L}_{car} + \bar{\triangle}_i)^2}{L_{s,i}} \dot{N}_i + \left[\beta_{i,i-1}\bar{v}_{i-1} - \bar{v}_i\right], i > 1 \qquad (3)$$

$$\dot{\bar{v}}_i = f(\bar{v}_i, \bar{\triangle}_i, \dot{\bar{\triangle}}_i) \qquad (4)$$

- The function, f on the right hand side embodies the aggregate driving behavior and is specific to a section.

Governing Equations

- Constitutive equations for car-following:

$$\dot{\bar{\triangle}}_i = \frac{(\bar{L}_{car} + \bar{\triangle}_i)^2}{L_{s,i}} \dot{N}_i + \left[\beta_{i,i-1}\bar{v}_{i-1} - \bar{v}_i\right], i > 1 \qquad (3)$$

$$\dot{\bar{v}}_i = f(\bar{v}_i, \bar{\triangle}_i, \dot{\bar{\triangle}}_i) \qquad (4)$$

- The function, f on the right hand side embodies the aggregate driving behavior and is specific to a section.

- Available measurements:
  - Number of vehicles passing through the detector location,
  - Average speed of vehicles that pass through the detector location, and
  - Occupancy.

US 183 Traffic Data

- Available measurements:
  - Number of vehicles passing through the detector location,
  - Average speed of vehicles that pass through the detector location, and
  - Occupancy.

US 183 Traffic Data

- Available measurements:
  - Number of vehicles passing through the detector location,
  - Average speed of vehicles that pass through the detector location, and
  - Occupancy.

US 183 Traffic Data

- Available measurements:
  - Number of vehicles passing through the detector location,
  - Average speed of vehicles that pass through the detector location, and
  - Occupancy.

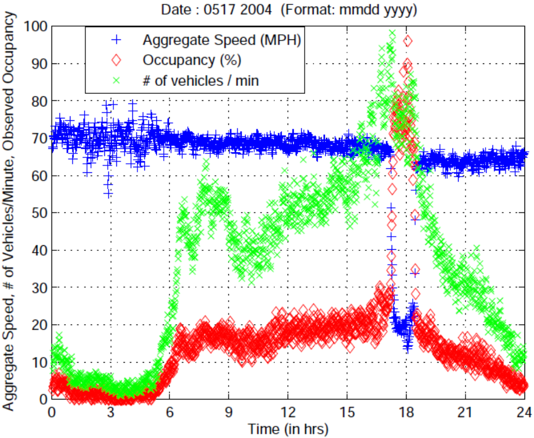

Figure : Traffic Data for 05/17/2004 at a location on US 183 NB (Metric Blvd.)

Rationale for a multi-regime aggregate car-following model

Traffic flow regimes: Different vehicle following behaviors can be hypothesized $\rightarrow$ structure of f.

- No interaction among vehicles: Free Regime. Typically from 10 PM to 7 AM. Not interesting
- Interaction among vehicles:
  - Regime 1: Observed from 7 AM to 10 PM except the time duration when there is a sharp decrease in traffic aggregate speed.

$$\dot{\bar{v}}_i = \frac{1}{h_w}\left[\dot{\bar{\Delta}}_i + \lambda_1(\bar{\Delta}_i - h_w \bar{v}_i\right] \tag{5}$$

  - Regime 2: Sharp decrease in traffic aggregate speed. Observed sometime between 4:30 PM to 6:30 PM. The duration of this regime is determined by observing the occupancy levels.

$$\dot{\bar{v}}_i = \frac{1}{\alpha p \bar{v}_i^{p-1}}\left[\dot{\bar{\Delta}}_i + \lambda_2(\bar{\Delta}_i - \alpha \bar{v}_i^{p}\right] \tag{6}$$

Rationale for a multi-regime aggregate car-following model

Traffic flow regimes: Different vehicle following behaviors can be hypothesized $\rightarrow$ structure of f.

- No interaction among vehicles: Free Regime. Typically from 10 PM to 7 AM. Not interesting
- Interaction among vehicles:
  - Regime 1: Observed from 7 AM to 10 PM except the time duration when there is a sharp decrease in traffic aggregate speed.

$$\dot{\bar{v}}_i = \frac{1}{h_w}\left[\dot{\bar{\triangle}}_i + \lambda_1(\bar{\triangle}_i - h_w\bar{v}_i\right] \tag{5}$$

  - Regime 2: Sharp decrease in traffic aggregate speed. Observed sometime between 4:30 PM to 6:30 PM. The duration of this regime is determined by observing the occupancy levels.

$$\dot{\bar{v}}_i = \frac{1}{\alpha p \bar{v}_i^{p-1}}\left[\dot{\bar{\triangle}}_i + \lambda_2(\bar{\triangle}_i - \alpha\bar{v}_i^p\right] \tag{6}$$

Rationale for a multi-regime aggregate car-following model

Traffic flow regimes: Different vehicle following behaviors can be hypothesized $\rightarrow$ structure of f.

- No interaction among vehicles: Free Regime. Typically from 10 PM to 7 AM. Not interesting
- Interaction among vehicles:
  - Regime 1: Observed from 7 AM to 10 PM except the time duration when there is a sharp decrease in traffic aggregate speed.

$$\dot{\bar{v}}_i = \frac{1}{h_w}\left[\dot{\bar{\triangle}}_i + \lambda_1(\bar{\triangle}_i - h_w \bar{v}_i\right] \tag{5}$$

  - Regime 2: Sharp decrease in traffic aggregate speed. Observed sometime between 4:30 PM to 6:30 PM. The duration of this regime is determined by observing the occupancy levels.

$$\dot{\bar{v}}_i = \frac{1}{\alpha p \bar{v}_i^{p-1}}\left[\dot{\bar{\triangle}}_i + \lambda_2(\bar{\triangle}_i - \alpha \bar{v}_i^p\right] \tag{6}$$

Rationale for a multi-regime aggregate car-following model

Traffic flow regimes: Different vehicle following behaviors can be hypothesized $\rightarrow$ structure of f.

- No interaction among vehicles: Free Regime. Typically from 10 PM to 7 AM. Not interesting
- Interaction among vehicles:
  - Regime 1: Observed from 7 AM to 10 PM except the time duration when there is a sharp decrease in traffic aggregate speed.

$$\dot{\bar{v}}_i = \frac{1}{h_w}\left[\dot{\bar{\triangle}}_i + \lambda_1(\bar{\triangle}_i - h_w \bar{v}_i\right] \tag{5}$$

  - Regime 2: Sharp decrease in traffic aggregate speed. Observed sometime between 4:30 PM to 6:30 PM. The duration of this regime is determined by observing the occupancy levels.

$$\dot{\bar{v}}_i = \frac{1}{\alpha p \bar{v}_i^{p-1}}\left[\dot{\bar{\triangle}}_i + \lambda_2(\bar{\triangle}_i - \alpha \bar{v}_i^p\right] \tag{6}$$

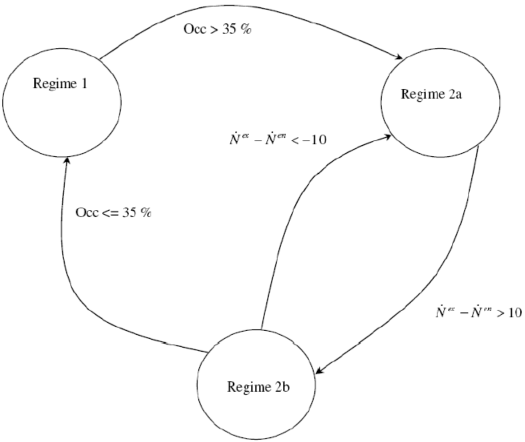

Figure : Schematic of the traffic regime transition

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Propagation of disturbances

- Breakdown in an upstream section:
  - Number of vehicles entering the downstream section is low
  - Occupancy is correspondingly small
  - Regime for vehicle-following changes to a less congested one.
- Breakdown in a downstream section:
  - Velocity drops in the downstream section.
  - Average following distance decreases in the upstream section.
  - Occupancy and regime changes correspondingly.

Corroboration of the model

- Parameter Identification: Parameters appear in a set of non-linear, ordinary differential equations and not all states are known.
  - Since speed is assumed to be a known quantity, parameters in the governing equations for the balance of the number of vehicles and for car-following can now be separately handled.
- Prediction: Since not all states are known (initially), one must estimate the initial condition.
  - A Kalman filter estimates the initial conditions of the unknown states asymptotically for any linear, completely observable system while minimizing an integral of a quadratic function of the estimation error of the state.
  - Extended Kalman filter is a generalization of Kalman filter with no guarantees but is reported to have excellent performance in practical applications.

Corroboration of the model

- Parameter Identification: Parameters appear in a set of non-linear, ordinary differential equations and not all states are known.
  - Since speed is assumed to be a known quantity, parameters in the governing equations for the balance of the number of vehicles and for car-following can now be separately handled.
- Prediction: Since not all states are known (initially), one must estimate the initial condition.
  - A Kalman filter estimates the initial conditions of the unknown states asymptotically for any linear, completely observable system while minimizing an integral of a quadratic function of the estimation error of the state.
  - Extended Kalman filter is a generalization of Kalman filter with no guarantees but is reported to have excellent performance in practical applications.

Corroboration of the model

- Parameter Identification: Parameters appear in a set of non-linear, ordinary differential equations and not all states are known.
  - Since speed is assumed to be a known quantity, parameters in the governing equations for the balance of the number of vehicles and for car-following can now be separately handled.
- Prediction: Since not all states are known (initially), one must estimate the initial condition.
  - A Kalman filter estimates the initial conditions of the unknown states asymptotically for any linear, completely observable system while minimizing an integral of a quadratic function of the estimation error of the state.
  - Extended Kalman filter is a generalization of Kalman filter with no guarantees but is reported to have excellent performance in practical applications.

Corroboration of the model

- Parameter Identification: Parameters appear in a set of non-linear, ordinary differential equations and not all states are known.
  - Since speed is assumed to be a known quantity, parameters in the governing equations for the balance of the number of vehicles and for car-following can now be separately handled.
- Prediction: Since not all states are known (initially), one must estimate the initial condition.
  - A Kalman filter estimates the initial conditions of the unknown states asymptotically for any linear, completely observable system while minimizing an integral of a quadratic function of the estimation error of the state.
  - Extended Kalman filter is a generalization of Kalman filter with no guarantees but is reported to have excellent performance in practical applications.

Corroboration of the model

- Parameter Identification: Parameters appear in a set of non-linear, ordinary differential equations and not all states are known.
    - Since speed is assumed to be a known quantity, parameters in the governing equations for the balance of the number of vehicles and for car-following can now be separately handled.
- Prediction: Since not all states are known (initially), one must estimate the initial condition.
    - A Kalman filter estimates the initial conditions of the unknown states asymptotically for any linear, completely observable system while minimizing an integral of a quadratic function of the estimation error of the state.
    - Extended Kalman filter is a generalization of Kalman filter with no guarantees but is reported to have excellent performance in practical applications.

Corroboration of the model

- Estimate traffic parameters from historic traffic data. Traffic parameters are location specific.
- Use estimated traffic parameters to predict traffic state in real time.
- Extended Kalman Filtering technique utilized for corroboration purposes.

Corroboration of the model

- Estimate traffic parameters from historic traffic data. Traffic parameters are location specific.
- Use estimated traffic parameters to predict traffic state in real time.
- Extended Kalman Filtering technique utilized for corroboration purposes.

Corroboration of the model

- Estimate traffic parameters from historic traffic data. Traffic parameters are location specific.
- Use estimated traffic parameters to predict traffic state in real time.
- Extended Kalman Filtering technique utilized for corroboration purposes.

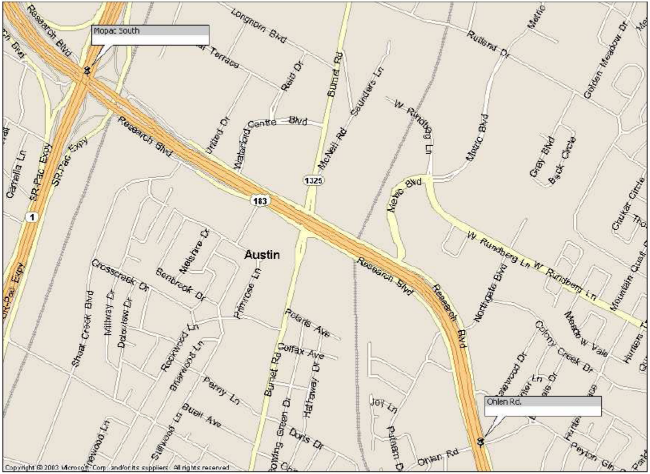

Figure : Map of the section selected for corroborating the model

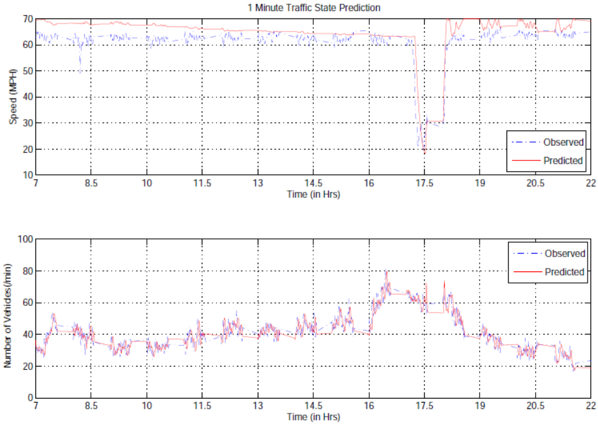

Figure : Prediction of the state of traffic for 05/10/2004

On-going work

- Corroboration of the model for a small network of sections.
- Real-time trip time prediction.

On-going work

- Corroboration of the model for a small network of sections.
- Real-time trip time prediction.