

Towards parametric model order reduction for nonlinear PDE systems in networks

Workshop on Model Reduction in Pilsen

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Outline

Motivation

PDAE-model

Finite Element Method

Simulation results

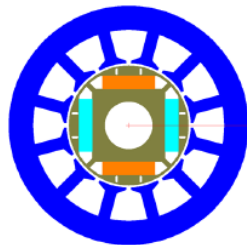
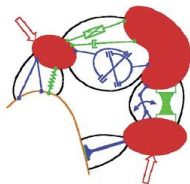
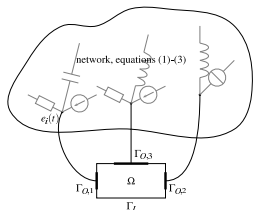
Construction of the reduced model

Location dependence of reduced model

Residual based parameter sampling

PABTEC and POD, joint work with A. Steinbrecher & Tatjana Stykel

Sketch: network with complex and simple components



L: EN (MoreSim4Nano), M: MBS (courtesy BMBF SOFA), R: PSM (BMBF SIMUROM)

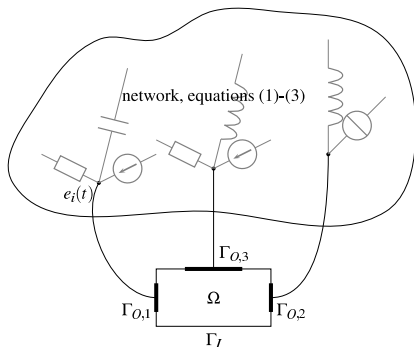
Modeling

- ▶ *PDAE system*

Aim

- ▶ Accurate reduced order models for complex components in networks
- ▶ Validity over relevant parameter range
- ▶ Accurate and robust *physical* reduced order model of the coupled system

Motivation: Coupled circuit and semiconductor models



Aim

- ▶ Accurate reduced order models for semiconductors in networks
- ▶ Validity over relevant parameter range
- ▶ Accurate *physical* reduced order model of the coupled system

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Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

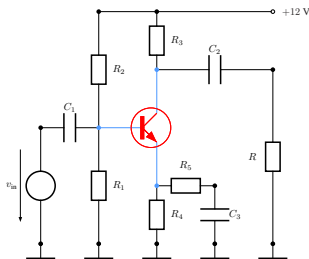
Kirchhoff's' laws (no semiconductors) read

$$A_j = 0, \quad v = A^T e$$

A : incidence matrix.

Voltage-current relations of components:

$$j_C = \frac{dq_C}{dt}(v_C, t), \quad j_R = g(v_R, t), \quad v_L = \frac{d\phi_L}{dt}(j_L, t)$$



Modified Nodal Analysis: join all equations to DAE system

$$A_C \frac{dq_C}{dt} (A_C^T e(t), t) + A_R g (A_R^T e(t), t) + A_L j_L(t) + A_V j_V(t) = -A_I i_s(t),$$

$$\frac{d\phi_L}{dt} (j_L(t), t) - A_L^T e(t) = 0,$$

$$A_V^T e(t) = v_s(t).$$

Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

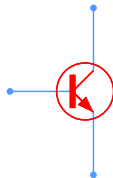
How can semiconductors be introduced?

- ▶ replace semiconductor by a (possibly nonlinear) electrical network,
- ▶ stamp semiconductor network into surrounding network,
- ▶ apply Modified Nodal Analysis.
- ▶ Here: use PDE model for semiconductors \rightarrow DD equations.

Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

PDE-model (drift-diffusion equations) for semiconductors

$$\begin{aligned} \operatorname{div}(\varepsilon \nabla \psi) &= q(n - p - C), \\ -q \partial_t n + \operatorname{div} J_n &= qR(n, p), \\ q \partial_t p + \operatorname{div} J_p &= -qR(n, p), \\ J_n &= \mu_n q (-U_T \nabla n - n \nabla \psi), \\ J_p &= \mu_p q (-U_T \nabla p - p \nabla \psi), \end{aligned}$$



on $\Omega \times [0, T]$ with $\Omega \subset \mathbb{R}^d$ ($d = 1, 2, 3$).

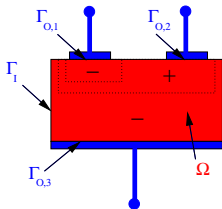
Dirichlet boundary constraints at $\Gamma_{O,k}$:

$$\psi(t, x) = \text{next slide}, \quad n(t, x) = \tilde{n}(x), \quad p(t, x) = \tilde{p}(x)$$

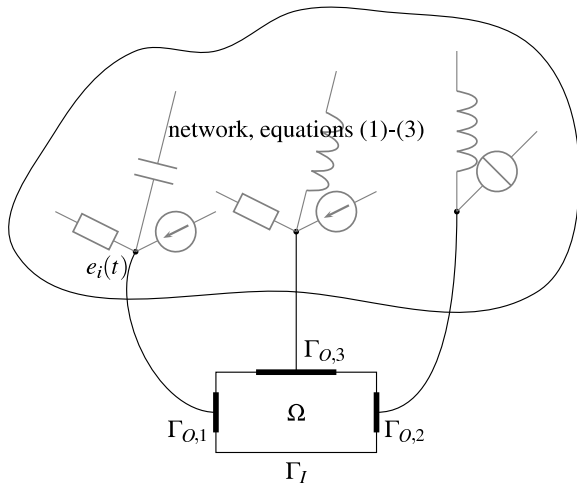
and Neumann boundary constraints at Γ_I :

$$\nabla \psi(t, x) \cdot \nu(x) = J_n \cdot \nu(x) = J_p(t, x) \cdot \nu(x) = 0$$

or mixed boundary conditions at MI contacts (MOSFETs).



Couple semiconductor to circuit [M. Günther '01, C. Tischendorf '03]



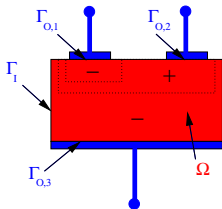
Couple semiconductor to circuit [M. Günther '01, C. Tischendorf '03]

Coupling conditions:

$$j_{S,k}(t) = \int_{\Gamma_{O,k}} (J_n + J_p - \varepsilon \partial_t \nabla \psi) \cdot \nu \, d\sigma,$$

$$\psi(t, x) = \psi_{bi}(x) + (A_S^T e(t))_k$$

for $(t, x) \in [0, T] \times \Gamma_{O,k}$,



and add current j_S to Kirchhoff's current law:

$$A_C \frac{dq_C}{dt} (A_C^T e, t) + A_{RG} (A_R^T e, t) + A_L j_L + A_V j_V + A_S j_S = -A_I i_S,$$

$$\frac{d\phi_L}{dt} (j_L, t) - A_L^T e = 0,$$

$$A_V^T e = v_S.$$

Add DD-equations + coupling conditions for each semiconductor.

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Mixed formulation

The electric field $E = -\nabla\psi$ plays dominant role in DD-equations.

Mixed formulation

[Brezzi et al. '05]

Provide additional variable g_ψ and equation

$$g_\psi = \nabla\psi.$$

Scaled DD equations then read:

$$\begin{aligned} \lambda \operatorname{div} g_\psi &= n - p - C, \\ -\partial_t n + \nu_n \operatorname{div} J_n &= R(n, p), \\ \partial_t p + \nu_p \operatorname{div} J_p &= -R(n, p), \\ g_\psi &= \nabla\psi, \\ J_n &= \nabla n - n g_\psi, \\ J_p &= -\nabla p - p g_\psi. \end{aligned}$$

Finite Element approximation

Finite elements

- ▶ piecewise constant ansatz functions for ψ , n and p .
Basis functions: φ_i , $i = 1, \dots, N$, $N = |\mathcal{T}|$.
- ▶ Raviart-Thomas elements for g_ψ , J_n and J_p .
Basis functions: ϕ_j , $i = 1, \dots, M$, $M = |\mathcal{E}| - |\mathcal{E}_N|$.

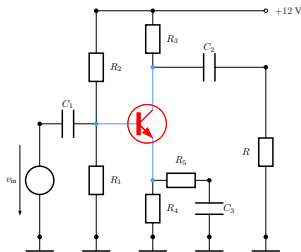
$$RT_0 := \{y : \Omega \rightarrow \mathbb{R}^d : y|_T(x) = a_T + b_T x, a_T \in \mathbb{R}^d, b_T \in \mathbb{R}, [y]_E \cdot \nu_E = 0, \text{ for all inner edges } E\}.$$

Galerkin ansatz:

$$\psi^h(t, x) = \sum_{i=1}^N \psi_i(t) \varphi_i(x), \quad g_\psi^h(t, x) = \sum_{j=1}^M g_{\psi,j}(t) \phi_j(x),$$

and analogously for n , p , J_n , and J_p .

Full model



$$\begin{aligned}
 A_C \frac{dq_C}{dt} (A_C^T e(t), t) + A_{RG} (A_R^T e(t), t) \\
 + A_{Lj_L}(t) + A_{Vj_V}(t) + A_S j_S(t) = -A_I i_S(t), \\
 \frac{d\phi_L}{dt} (j_L(t), t) - A_L^T e(t) = 0, \\
 A_V^T e(t) = v_S(t),
 \end{aligned}$$

$$j_S(t) - C_1 j_n(t) - C_2 j_p(t) - C_3 \dot{g}_\psi(t) = 0,$$

$$\begin{pmatrix} 0 \\ -M_L \dot{n}(t) \\ M_L \dot{p}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} \psi(t) \\ n(t) \\ p(t) \\ g_\psi(t) \\ j_n(t) \\ j_p(t) \end{pmatrix} + \mathcal{F}(n^h, p^h, g_\psi^h) - b(A_S^T e(t)) = 0.$$

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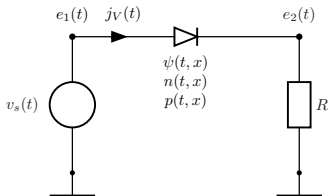
Construction of the reduced model

Location dependence of reduced model

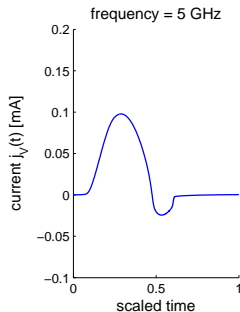
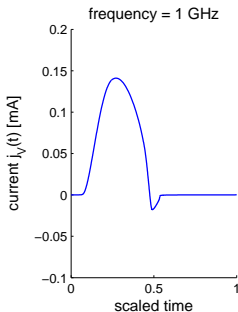
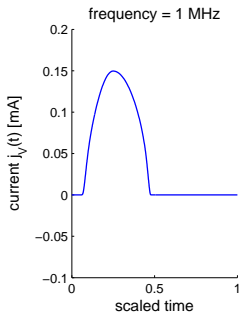
Residual based parameter sampling

PABTEC and POD, joint work with A. Steinbrecher & Tatjana Stykel

Basic test circuit, simulation results



input voltage: $v_s(t) = 5[V] \cdot \sin(2\pi f \cdot t)$
 similar results obtained by MECS [Selva Soto]



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Snapshot-POD (Proper Orthogonal Decomposition) [L. Sirovich '87]

Full simulation yields snapshots (here: $y = \psi, n, p, \dots$)

$$\{y(t_i, \cdot)\}_{i=1, \dots, m} \subset \text{span}\{\varphi_j\}_{j=1, \dots, N}, \quad \text{with} \quad y(t_i, x) = \sum_{j=1}^N \tilde{y}_j(t_i) \varphi_j(x).$$

Gather coefficients in matrix

$$Y := (\vec{y}(t_1), \dots, \vec{y}(t_m)) \in \mathbb{R}^{N \times m}.$$

POD in Hilbert space X as eigenvalue problem:

$$Kv^k = \sigma_k^2 v^k, \quad \text{with} \quad K_{ij} := \langle y(t_i, \cdot), y(t_j, \cdot) \rangle_X.$$

Note that $K = Y^T M Y$ with $M_{ij} = \langle \varphi_i, \varphi_j \rangle_X$. Write POD in terms of SVD:

$$\tilde{U} \Sigma \tilde{V}^T = L^T Y, \quad \text{with} \quad LL^T := M.$$

Then, the s -dimensional POD basis is

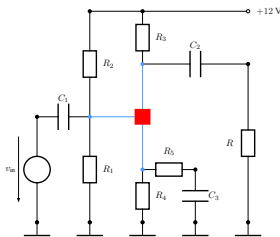
$$\left\{ u^i := \sum_{j=1}^N \tilde{u}_j^i \varphi_j(\cdot) \right\}_{i=1, \dots, s}, \quad U := (\tilde{u}^1, \dots, \tilde{u}^s) := L^{-T} \tilde{U}_{(:, 1:s)}.$$

Model Order Reduction

- ▶ Simulate the complete network at one or more reference parameters.
- ▶ Take snapshots of the state of each semiconductor at time points t_i .
- ▶ Perform POD **component wise** on ψ , n , p , g_ψ , J_n and J_p .
- ▶ Use the POD basis functions as (non local) Galerkin ansatz functions:

$$\psi^{POD}(t, x) = \sum_{i=1}^s \gamma_{\psi,i}(t) u_{\psi}^i(x)$$

Reduced model



$$\begin{aligned}
 A_C \frac{dq_C}{dt} (A_C^T e(t), t) + A_{Rg} (A_R^T e(t), t) \\
 + A_L j_L(t) + A_V j_V(t) + A_S j_S(t) = -A_I i_S(t), \\
 \frac{d\phi_L}{dt} (j_L(t), t) - A_L^T e(t) = 0, \\
 A_V^T e(t) = v_S(t),
 \end{aligned}$$

$$j_S(t) - C_1 U_{J_n} \gamma_{J_n}(t) - C_2 U_{J_p} \gamma_{J_p}(t) - C_3 U_{g_\psi} \dot{\gamma}_{g_\psi}(t) = 0,$$

$$\begin{pmatrix} 0 \\ -\dot{\gamma}_n(t) \\ \dot{\gamma}_p(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{POD} \begin{pmatrix} \gamma_\psi(t) \\ \gamma_n(t) \\ \gamma_p(t) \\ \gamma_{g_\psi}(t) \\ \gamma_{J_n}(t) \\ \gamma_{J_p}(t) \end{pmatrix} + U^T \mathcal{F}(n^{POD}, p^{POD}, g_\psi^{POD}) - U^T b(A_S^T e(t)) = 0.$$

Computational complexity

Computational complexity of reduced model still depends on n_{FEM} :

$$U^T \mathcal{F}(n^{POD}, p^{POD}, g_{\psi}^{POD}) = \underbrace{U^T}_{n_{POD} \times n_{FEM}} \underbrace{F}_{n_{FEM}} \left(\underbrace{U_n}_{n_{FEM} \times n_{POD}}, \gamma_n, U_p \gamma_p, U_{g_{\psi}} \gamma_{g_{\psi}} \right).$$

With matrix-matrix multiplications in Jacobian computation:

$$\underbrace{U^T}_{n_{POD} \times n_{FEM}, \text{ block-dense}} \underbrace{F'(\dots)}_{n_{FEM} \times n_{FEM}, \text{ sparse}} \underbrace{U}_{n_{FEM} \times n_{POD}, \text{ block-dense}}.$$

Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen '09]

DEIM

- ▶ Do POD on snapshots $\{F(n(t_i), p(t_i), g_{\psi}(t_i))\}$, obtain basis $W \in \mathbb{R}^{n_{FEM} \times n_{DEIM}}$ (block diagonal matrix).

- ▶ Ansatz

$$F(U_n \gamma_n(t), U_p \gamma_p(t), U_{g_{\psi}} \gamma_{g_{\psi}}(t)) \approx Wc(t)$$

is overdetermined.

- ▶ Select n_{DEIM} “useful” rows:

$$P^T F(\dots) \approx P^T Wc(t).$$

- ▶ If $P^T W$ is regular:

$$F(\dots) \approx Wc(t) = W(P^T W)^{-1} P^T F(\dots)$$

The regularity of $P^T W$ can be guaranteed, see [CS09].
 Again we apply the method **component-wise**.

Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen '09]

Reduced model

$$U^T F(U_n \gamma_n, U_p \gamma_p, U_{g_\psi} \gamma_{g_\psi})$$

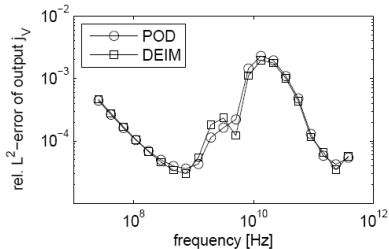
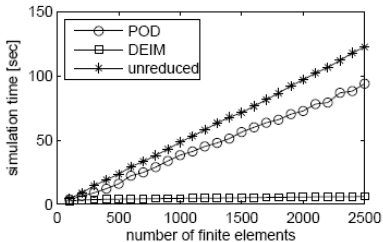
with DEIM:

$$\underbrace{(U^T W (P^T W)^{-1})}_{n_{POD} \times n_{DEIM}, \text{ block-dense}} \underbrace{P^T F(U_n \gamma_n, U_p \gamma_p, U_{g_\psi} \gamma_{g_\psi})}_{\substack{n_{DEIM} \\ n_{FEM}}}$$

Results for 1D-diode:

	n_{FEM}	FEM	n_{POD}	ROM	n_{DEIM}	ROM + DEIM
	3003	3.15 sec.	220	3.52 sec.	187	1.93 sec.
	15009	23.5 sec.	229	19.9 sec.	198	4.04 sec.
	48015	82.3 sec.	229	74.2 sec.	199	9.87 sec.
order		$\approx n_{FEM}^{1.18}$		$\approx n_{FEM}^{1.10}$		$\approx n_{FEM}^{0.578}$

Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen '09]



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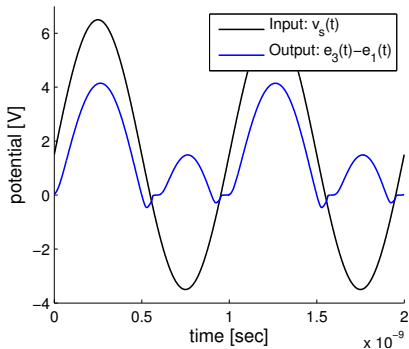
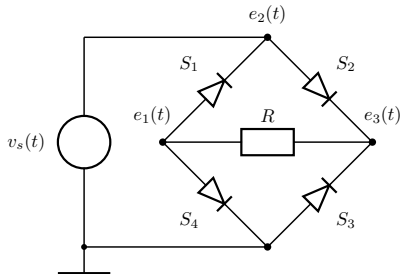
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Reduced model depends on position of diode in network

Bridge rectifier with 4 diodes:



Reduced model depends on position of diode in network

The distance between the spaces U^1 and U^2 which are spanned, e.g., by the POD-functions U_{ψ}^1 of the diode S_1 and U_{ψ}^2 of the diode S_2 respectively, is measured by

$$d(U^1, U^2) := \max_{\substack{u \in U^1 \\ \|u\|_2=1}} \min_{\substack{v \in U^2 \\ \|v\|_2=1}} \|u - v\|_2 = \sqrt{2 - 2\sqrt{\lambda}},$$

where λ is the smallest eigenvalue of the positive definite matrix SS^T with $S_{ij} = \langle u_{\psi,i}^1, u_{\psi,j}^2 \rangle_2$.

Δ	$d(U^1, U^2)$	$d(U^1, U^3)$
10^{-4}	0.61288	$5.373 \cdot 10^{-8}$
10^{-5}	0.50766	$4.712 \cdot 10^{-8}$
10^{-6}	0.45492	$2.767 \cdot 10^{-7}$
10^{-7}	0.54834	$1.211 \cdot 10^{-6}$

Table: Distances between reduced models in the rectifier network.

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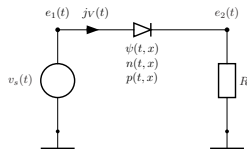
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Problem setting

MOR test problem

Basic circuit with **frequency f** of the voltage source $v_s(t) = 5[V] \cdot \sin(2\pi f \cdot t)$
 as **model parameter**.

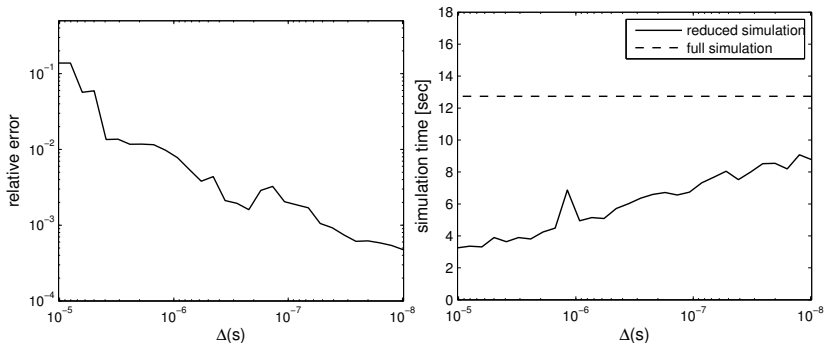


Lack of information

Select number of snapshots so that $\Delta(s) = \sqrt{\frac{\sum_{i=s+1}^m \sigma_i^2}{\sum_{i=1}^m \sigma_i^2}} \approx \text{tol}$.

Reduced model at a fixed frequency

First test: Compare reduced and unreduced system at a fixed frequency.

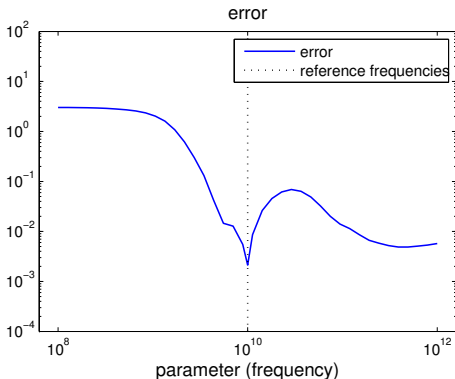


Reduced model over parameter space

Construction of reduced model requires snapshots from full simulations at reference parameters.

Is the model valid over a large parameter space?

reference parameter: $P_1 := \{f_1\} := \{10^{10}[\text{Hz}]\}$
 parameter space $\mathcal{P} = [10^8, 10^{12}]$



Reduced model over parameter space - sampling

Goal

Find new sampling parameter f_{k+1} (reference frequency) without simulating the full, unreduced system. Set $P_{k+1} := P_k \cup \{f_{k+1}\}$.

- ▶ We do not consider the PDE discretization error.
- ▶ Rigorous upper bound for the error not available

$$\|\mathcal{E}(f; P_k)\| = \|y^h(f) - y^{POD}(f; P_k)\| \leq ?(s)$$

where $y^h := (\psi^h, n^h, p^h, g_\psi^h, J_n^h, J_p^h)^\top$, $y^{POD} := (\psi^{POD}, n^{POD}, \dots)^\top$.

- ▶ Rigorous RB methods, Greedy algorithm [see e.g. A. Patera, G. Rozza '07]: a-posteriori error estimates required.
- ▶ Linear ODEs [see e.g. B. Haasdonk, M. Ohlberger '09]: build difference between residual and unreduced equation to derive an ODE for the error.

Residual based sampling

Define residual $\mathcal{R}(z^{POD}(f; P_k))$: insert $z^{POD}(f; P_k)$ into unreduced equation,

$$\mathcal{R} := \begin{pmatrix} 0 \\ -M_L \dot{n}^{POD}(t) \\ M_L \dot{p}^{POD}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} \psi^{POD}(t) \\ n^{POD}(t) \\ p^{POD}(t) \\ g_{\psi}^{POD}(t) \\ J_n^{POD}(t) \\ J_p^{POD}(t) \end{pmatrix} + \mathcal{F}(n^{POD}, p^{POD}, g_{\psi}^{POD}) - b(e^{POD}(t)).$$

Residual admits different scales.

Scale with block diagonal matrix-valued function

$$D(f) := \text{diag}(d_{\psi}(f)I, d_n(f)I, d_p(f)I, d_{g_{\psi}}(f)I, d_{J_n}(f)I, d_{J_p}(f)I)$$

and choose $d_{\psi}(f)$ according to

$$d_{\psi}(f_j) \cdot \|\mathcal{R}_{\psi}(y^{POD}(f_j; P_k))\| = \frac{\|\psi^h(f_j) - \psi^{POD}(f_j; P_k)\|}{\|\psi^h(f_j)\|}, \quad \forall f_j \in P_k.$$

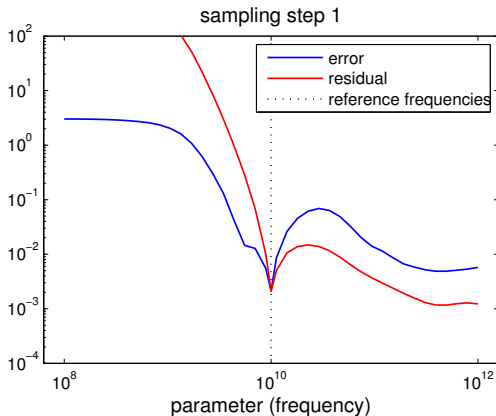
Residual based sampling

Algorithm: sampling

1. Select $f_1 \in \mathcal{P}$, $P_{test} \subset \mathcal{P}$, $tol > 0$, and set $k := 1$, $P_1 := \{f_1\}$.
2. Simulate the unreduced model at f_1 and calculate the reduced model with POD basis functions U_1 .
3. Calculate weight functions $d_{(\cdot)}(f) > 0$ for all $f \in P_k$.
4. Calculate the scaled residual $\|D(f)\mathcal{R}(z^{POD}(f, P_k))\|$ for all $f \in P_{test}$.
5. Check termination conditions, e.g.
 - ▶ $\max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_k))\| < tol$,
 - ▶ no progress in weighted residual.
6. Calculate $f_{k+1} := \arg \max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_k))\|$.
7. Simulate the unreduced model at f_{k+1} and create a new reduced model with POD basis U_{k+1} using also the already available information at f_1, \dots, f_k .
8. Set $P_{k+1} := P_k \cup \{f_{k+1}\}$, $k := k + 1$ and goto 3.

Numerical example - sampling step 1

Let $f_1 := 10^{10}[\text{Hz}]$, $P_1 := \{10^{10}[\text{Hz}]\}$, $\mathcal{P} = [10^8, 10^{12}]$.

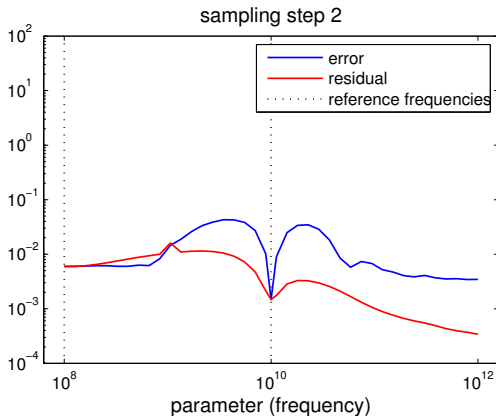


$$f_2 = \arg \max_{f \in P_{\text{test}}} \|D(f)\mathcal{R}(z^{\text{POD}}(f, P_1))\| = 10^8[\text{Hz}]$$

$$P_2 = \{10^8[\text{Hz}], 10^{10}[\text{Hz}]\}$$

Numerical example - sampling step 2

$$P_2 = \{10^8[\text{Hz}], 10^{10}[\text{Hz}]\}$$

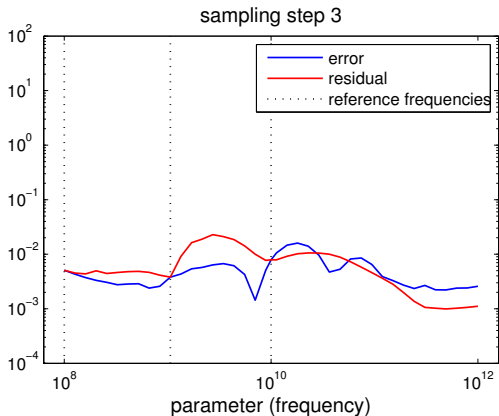


$$f_3 = \arg \max_{f \in P_{\text{test}}} \|D(f)\mathcal{R}(z^{\text{POD}}(f, P_2))\| = 1.0608 \cdot 10^9[\text{Hz}]$$

$$P_3 = \{10^8[\text{Hz}], 1.0608 \cdot 10^9[\text{Hz}], 10^{10}[\text{Hz}]\}$$

Numerical example - sampling step 3

$$P_3 = \{10^8[\text{Hz}], 1.0608 \cdot 10^9[\text{Hz}], 10^{10}[\text{Hz}]\}$$



Terminate with “no progress in residual”.

Outline

Motivation

PDAE-model

Finite Element Method

Simulation results

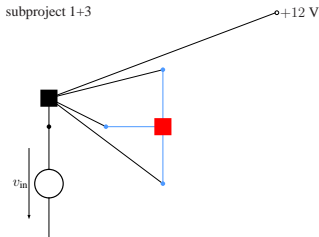
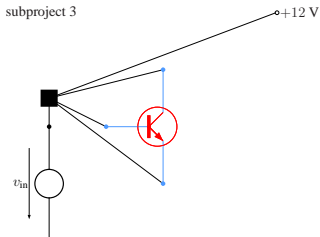
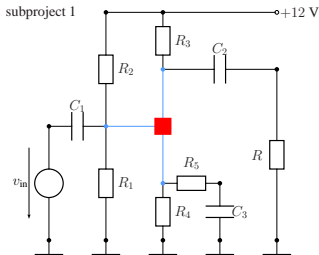
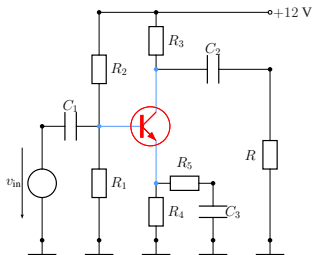
Construction of the reduced model

Location dependence of reduced model

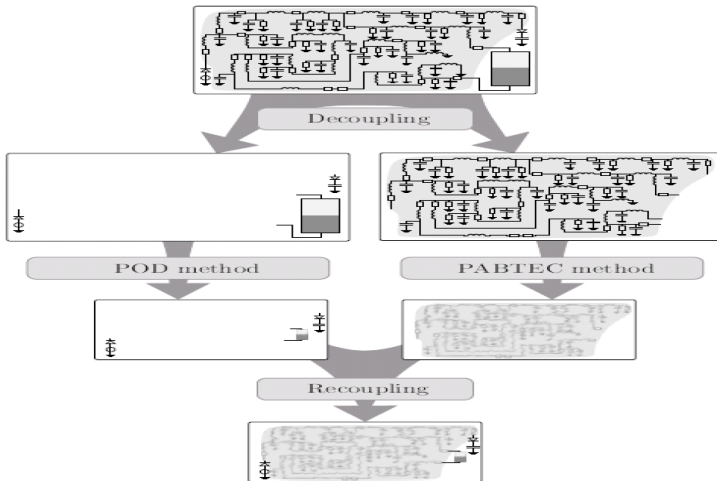
Residual based parameter sampling

PABTEC and POD, joint work with A. Steinbrecher & Tatjana Stykel

Combination of PABTEC (Reis & Stykel 2010) and POD; joint work with [A. Steinbrecher, T. Stykel]



Combination of PABTEC (Reis & Stykel '10) and POD



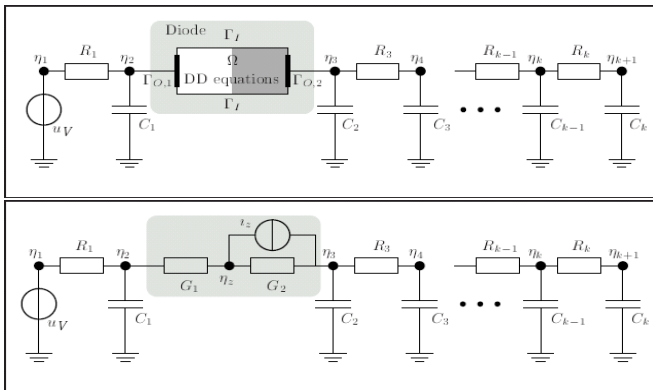
Substitution of nonlinear components for PABTEC and recoupling

A. Steinbrecher, T. Stykel (Int. J. Circuits Theory Appl., 2012):

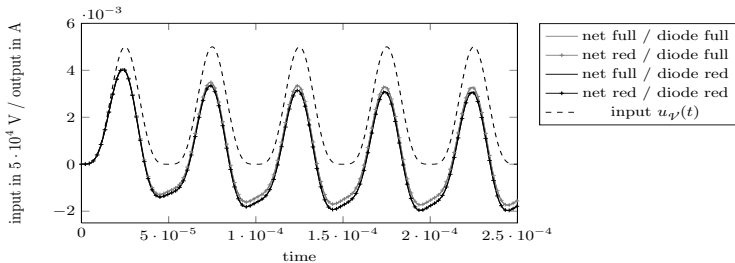
Nonlinear inductor \rightarrow current source

Nonlinear capacitor \rightarrow voltage source

Nonlinear resistor \rightarrow linear circuit with 2 serial resistors and one voltage source parallel to one of the resistors



Combination of PABTEC and POD; Int. J. Numer. Model. 2012



network (MNA equations)		diode (DD equations)		coupled system dim.	simul. time	Jacobian evaluations	absolute error $\ y - \hat{y}\ _{L_2}$	relative error $\frac{\ y - \hat{y}\ _{L_2}}{\ y\ _{L_2}}$
type	dim.	type	dim.					
unreduced	1503	unreduced	6006	7510	23.37s	20		
reduced	24	unreduced	6006	6031	16.90s	17	$2.165 \cdot 10^{-8}$	$7.335 \cdot 10^{-4}$
unreduced	1503	reduced	105	1609	1.51s	16	$2.952 \cdot 10^{-6}$	$1.000 \cdot 10^{-1}$
reduced	24	reduced	105	130	1.19s	11	$2.954 \cdot 10^{-6}$	$1.000 \cdot 10^{-1}$

Some literature

- M.H., M. Kunkel, U. Matthes, M. Vierling: Model order reduction of integrated circuits in electrical networks. System Reduction for Nanoscale IC Design Hamburger, Peter Benner (Ed), Mathematics in Industry 20, 2014.
- A. Steinbrecher, T. Stykel: Model order reduction of nonlinear circuit equations. Internat. J. Circuit Theory Appl., 41, 2013, pp.1226-1247.
- M.H., M. Kunkel, A. Steinbrecher, T. Stykel: Model order reduction of coupled circuit-device systems. Int. J. Numer. Model. 25:362-377 (2012).
- M.H., M. Kunkel: Residual Based Sampling in POD Model Order Reduction of Drift-Diffusion Equations in Parametrized Electrical Networks. ZAMM 92:91-104 (2012).
- M.H., U. Matthes: Model order reduction for networks of ODE and PDE systems. D. Hömberg and F. Tröltzsch (Eds.): CSMO 2011, IFIP AICT 391, pp. 92-101, 2013.
- M.H., M. Kunkel, M. Vierling: POD Model Order Reduction of Drift-Diffusion Equations in Electrical Networks. Lecture Notes in Electrical Engineering, Vol. 74, (2011).
- M.H., M. Kunkel, U. Matthes: POD Model Order Reduction of electrical networks with semiconductors modeled by the transient Drift-Diffusion equations. Proceedings of the ECMI Wuppertal 2010 (2010).
- M.H., M. Kunkel: Discrete empirical interpolation in POD Model Order Reduction of Drift-Diffusion Equations in Electrical Networks. SCEE Proceedings 2010, Toulouse (2010).

Next steps

- ▶ Include QDD models.
- ▶ Include EM effects.
- ▶ Apply approach to other equation networks containing simple and complex components.

Thank you for attending!



The work reported in this talk is supported by the German Federal Ministry of Education and Research (BMBF), grants 03HIPAE5 & 03MS613D.



MoreSim_4_Nano

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