Non-linear flows in porous media and application in engineering

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Outline

1. Motivation, Engineering Problems
2. Mathematical framework - Linear Darcy, Non-linear: Pre-Darcy, and Post-Darcy flows
3. IBVP, Boundary Data and Diffusive Capacity
4. Analytic formula accounting mixed different regimes of the flow
5. Productivity Index for Compressible Gas
6. Proof of Asymptotic Convergence of Transient Diffusive Capacity

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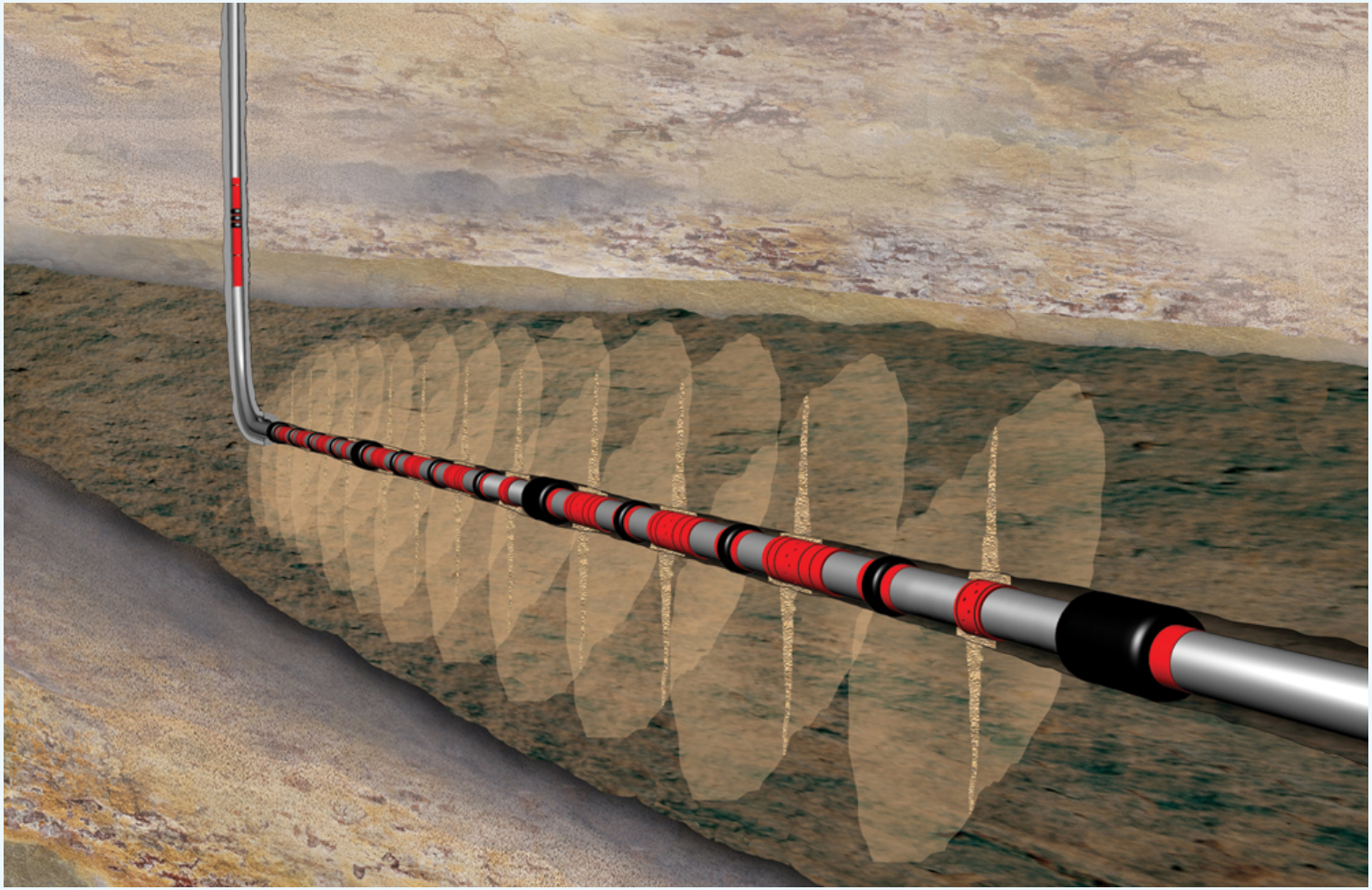
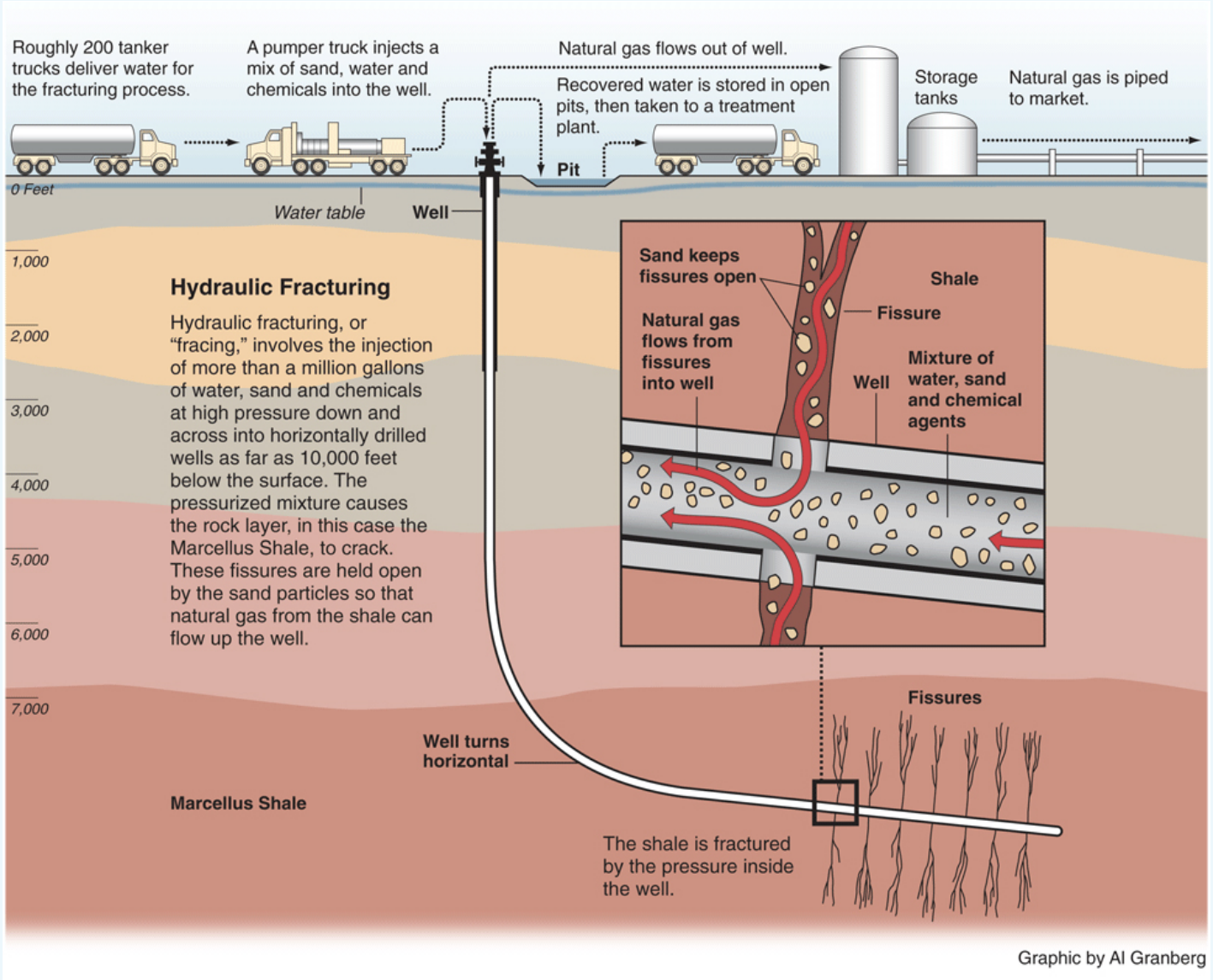
Fractured reservoir



Halliburtons RapidFrac® system

Real-life reservoir



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Hydrodynamical Parameters

Consider the flow of slightly compressible fluid in porous media

<u>Fluid:</u>

$$\rho(p) \;-\; \text{density}$$
$$1/\kappa \;-\; \text{compressibility} \;\sim\; 10^{-8} m^2/N$$
$$\mu \;-\; \text{viscosity}$$

<u>Porous media:</u> $k \;-\;$ permeability

<u>Flow:</u>

$$u(x,t) \;-\; \text{velocity}$$
$$p(x,t) \;-\; \text{pressure}$$

Linear and Nonlinear Flow

- Darcy's law (1856): $a_0 u = -\nabla p$

Post Darcy(Fast Flows)

- 2-Forchheimer law (1901): $a_0 u + \beta |u| u = -\nabla p$
- 3-Forchheimer law (1901): $a_0 u + a_1 |u| u + a_2 |u|^2 u = -\nabla p$
- Forchheimer power law (1930): $a_0 u + a_1 |u|^{m-1} u = -\nabla p$
  $1.6 \leq m \leq 2$

Pre-Darcy, slow flow, suggested 70-th

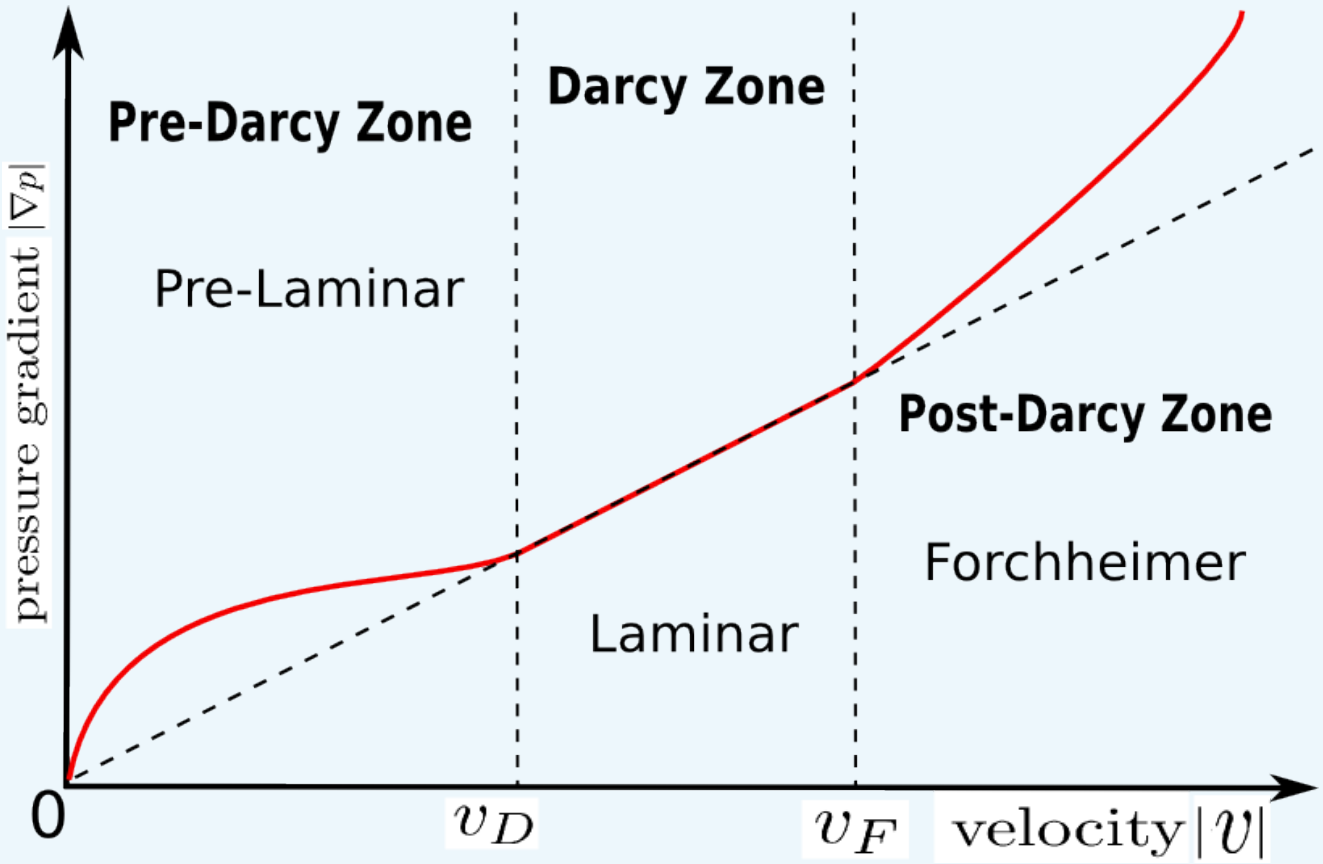
- Experiment by M. Soliman and his student F. Siddiqui at Tech, 2014

$$a_1 |u|^{-\alpha(x)} u = -\nabla p, \ 0 \leq \alpha(x) < 1$$

Generalization:

- In our group at Tech (from 2007): $g(|u|) u = -\nabla p$

Deviation from Darcy's law



$$-\nabla p = \begin{cases} \lambda |v|^{-s} v, \ 0 \le s < 1, & 0 \le |v| \le v_D, \quad \text{pre-Darcy flow;} \\ \alpha v, & v_D \le |v| \le v_F, \quad \text{Darcy flow;} \\ (\alpha + \beta |v|) v, & |v| \ge v_F, \quad \text{2-Forchheimer flow.} \end{cases}$$

Generalized g-Forchheimer equation

Generalized g-Forchheimer (momentum) equation:

$$\boxed{g(|u|)u = -\nabla p}$$

Function $g(s)$ is a GPPC:

$$g(s) = a_0 + \sum_{j=1}^{k} a_j s^{\alpha_j} = a_0 + a_1 s^{\alpha_1} + a_2 s^{\alpha_2} + \ldots + a_k s^{\alpha_k}$$

$$a_j \geq 0, \quad j = 1, \ldots, k, \quad a_0 > 0, \quad \alpha_j < \alpha_{j+1}, \quad j \geq 1$$

Two stages of the research:

First stage([Aulisa, Boshanskaya,Hoang,I,09; Hoang,I,10; Aulisa,Bloshanskaya,I,10;Hoang,I,11;Aulisa,Bloshanskaya,I,12]):

degree condition $\alpha_k = \deg(g) \leq \dfrac{4}{n-2}$

Second stage([Hoang,I,Kieu,Zeev, 13; Hoang,Kiue,Phan 14]):

non-degree condition $\alpha_k = \deg(g) > \dfrac{4}{n-2}$

Two equivalent forms of the nixed phases of the filtration

$$G(|u|)u = -\nabla p \qquad \Longleftrightarrow \qquad u = -K(|\nabla p|)\nabla p$$

generalized (nonlinear) Darcy equation

Mixed Phases of the Filtration in Porous Media

$$K(|\xi|) = \lambda^{-\frac{1}{1-s}}|\xi|^{\frac{s}{1-s}}, \quad 0 \le s < 1, \quad \text{pre-Darcy flow;} \tag{1}$$

$$K(|\xi|) = \alpha^{-1} \quad \text{Darcy flow;} \tag{2}$$

$$K(|\xi|) = \frac{2}{\alpha + \sqrt{\alpha^2 + 4\beta|\xi|}}, \quad \text{post-Darcy (Forchheimer) flow.} \tag{3}$$

In general K can be modeled as function K_*:

$$K_*(\xi) = \frac{\xi^{\beta_1}}{(1+\xi)^{\beta_1+\beta_2}} \quad \text{for } \xi \ge 0, \tag{4}$$

Parabolic equation for $p(x,t)$

Taking into account dependence of the coefficients on density ρ:

- Modified mixed equation with a density: $u = -K(\rho|\nabla p|)\nabla p$
- the continuity equation: $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho u)$
- the equation of state:
  $\rho = \rho_0 \exp\left(\frac{p-p_0}{\kappa}\right)$ $\qquad$ $\rho = Mp$

  Slightly comp. fluid $\qquad$ Ideal Gas

$$\boxed{\frac{1}{\kappa} \cdot \frac{\partial p}{\partial t} = \nabla \cdot (K(|\nabla p|)\nabla p)} \qquad \boxed{\frac{1}{\kappa} \cdot \frac{\partial p}{\partial t} = \nabla \cdot (K(|\nabla p^2|)\nabla p^2)}$$

For simplicity $\quad \kappa = 1 = M$

Main Structural properties of $K(\nabla p)$

Function $K(\xi)$, $\xi \in [0, \infty)$ is satisfies

$$-\beta_2 \frac{K(\xi)}{\xi} \leq K'(\xi) \leq \beta_1 \frac{K(\xi)}{\xi} \tag{5}$$

$K(|y|)y$, $y \in \mathbb{R}^n$, is monotone, i.e.

$$\big(K(|y'|)y' - K(|y|)y\big) \cdot (y' - y) \geq \frac{d_5 |y - y'|^{2+\beta_1}}{(1 + |y| + |y'|)^{\beta_1 + \beta_2}} \quad \forall y, y' \in \mathbb{R}^n, \tag{6}$$

where

$$d_5 = \frac{d_2(1 - \beta_2)}{2^{\beta_1 + 1}(\beta_1 + 1)}. \tag{7}$$

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Prescribed Flux or Pressure on the Boundary for Slightly Compressible Fluid Flows

$$\frac{\partial p}{\partial t} = \nabla \cdot (K(|\nabla p|)\nabla p) \text{ in} U$$

on Γ_i

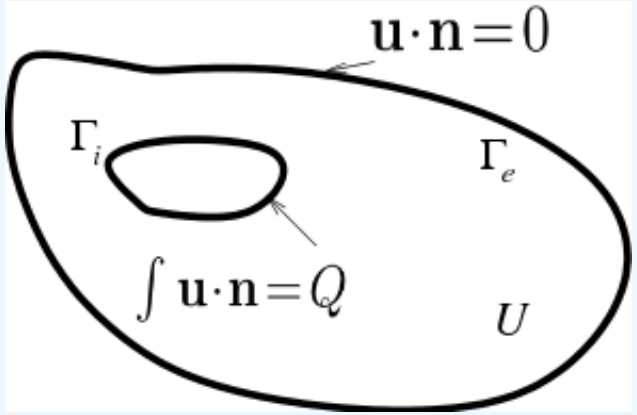
$$-K(|\nabla p|)\nabla p \cdot N = q(x,t) \text{ Neumann BC}$$

or

$$p(x,t) = \Psi(x,t) \text{ Dirichlet BC}$$

$$\left.\frac{\partial p}{\partial N}\right|_{\Gamma_e} = 0 \text{ Non-flow BC}$$

$$p(x,0) = p_0(x)$$



Well Productivity Index for Slightly Compressible Fluid Flows

Engineering parameters of well capacity *Productivity Index (PI) at time of observation*

$$J_g(\Gamma_i)(t) = PI = \frac{\text{Hydrocarbon Production}}{\text{Reservoir Pressure} - \text{Well Pressure}} = \frac{Q(t)}{PDD(t)}$$

Averages over the domain and on the boundary

$$\overline{p}_U(t) = \frac{1}{|U|}\int_U p(x,t)\,dx \qquad \overline{p}_{\Gamma_i}(t) = \frac{1}{|\Gamma_i|}\int_{\Gamma_i} p(x,t)\,ds$$

$PDD(t) = \overline{p}_U(t) - \overline{p}_{\Gamma_i}(t)$ *pressure drawdown* in the domain U

$Q(t) = \int_{\Gamma_i} u \cdot N\,ds$ *total flux* through Γ_i

<u>Important feature of the *PI*</u>: *During long time of the production with constant $Q = const$, productivity index $J_g(\Gamma_i)(t)$ is almost time invariant and for this reason is used by engineers to evaluate well capacity.*

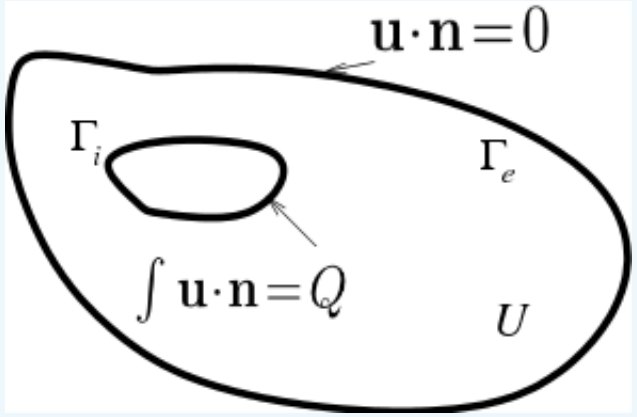
Question-For Which $p(x,0) = p_0(x)$ Initial Pressure $PI = const$ for all time ?

Case: Given Total Flux on the Well and no Flow on Reservoir Boundary.

$$\frac{\partial p}{\partial t} = \nabla \cdot (K(|\nabla p|)\nabla p)$$

$$-\int_{\Gamma_i} K(|\nabla p|)\nabla p \cdot N \, ds = Q(t)$$

$$p(x,0) = p_0(x)$$



No uniqueness $\Longrightarrow$ additional constraints on the boundary data.

PSS Profile, Engineering Theorem

PSS regime which characterizes by two parameters on the boundary:
I. Given pressure profile $\varphi_0(x)$, and II. Total flux Q_s constant.
These two parameters defines the PSS profile/regime:

$$p_s(x,t) = -\frac{Q_s}{|U|}t + W(x)$$

basic profile $W(x)$ is a solution of BVP with no flux on reservoir boundary

$$-A = \frac{Q_s}{|U|} = \nabla \cdot (K(|\nabla W|)\nabla W)$$
$$W(x)|_{\Gamma_i} = \varphi_0(x)$$

Theorem

If initial pressure $p(x,0) = W(x)$, *then* $PI = \frac{Q}{\int W(x)dx}$

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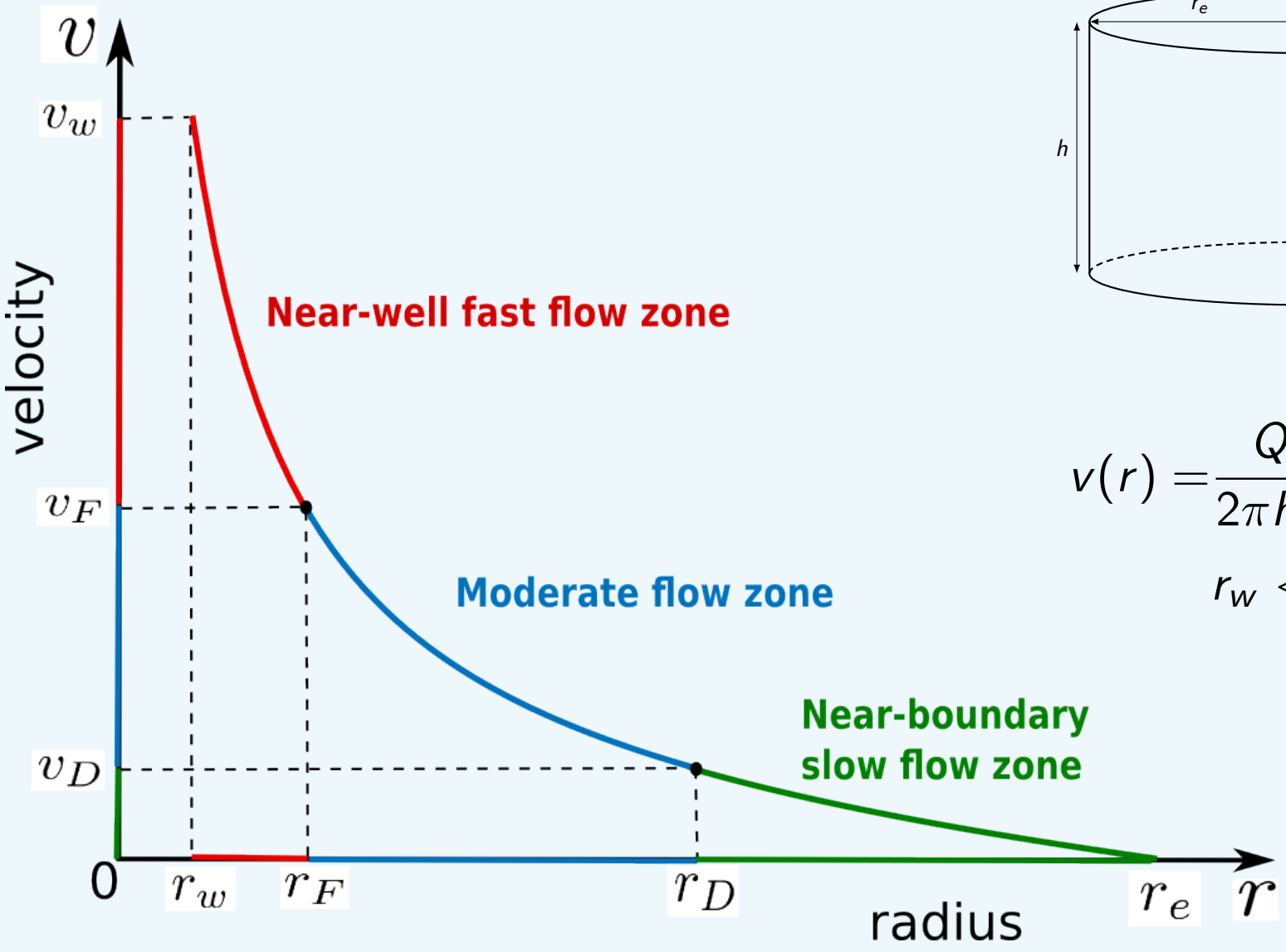
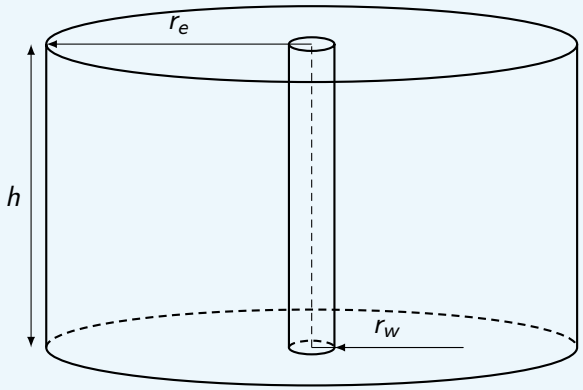
Axial-symmetric case





$$v(r) = \frac{Q(r_e^2 - r^2)}{2\pi h r(r_e^2 - r_w^2)}$$

$$r_w < r < r_e$$

Denote $$A = \frac{Q}{|U|} = \frac{Q}{2\pi h(r_e^2 - r_w^2)}, \quad L = 2\pi h(r_e^2 - r_w^2)^2$$

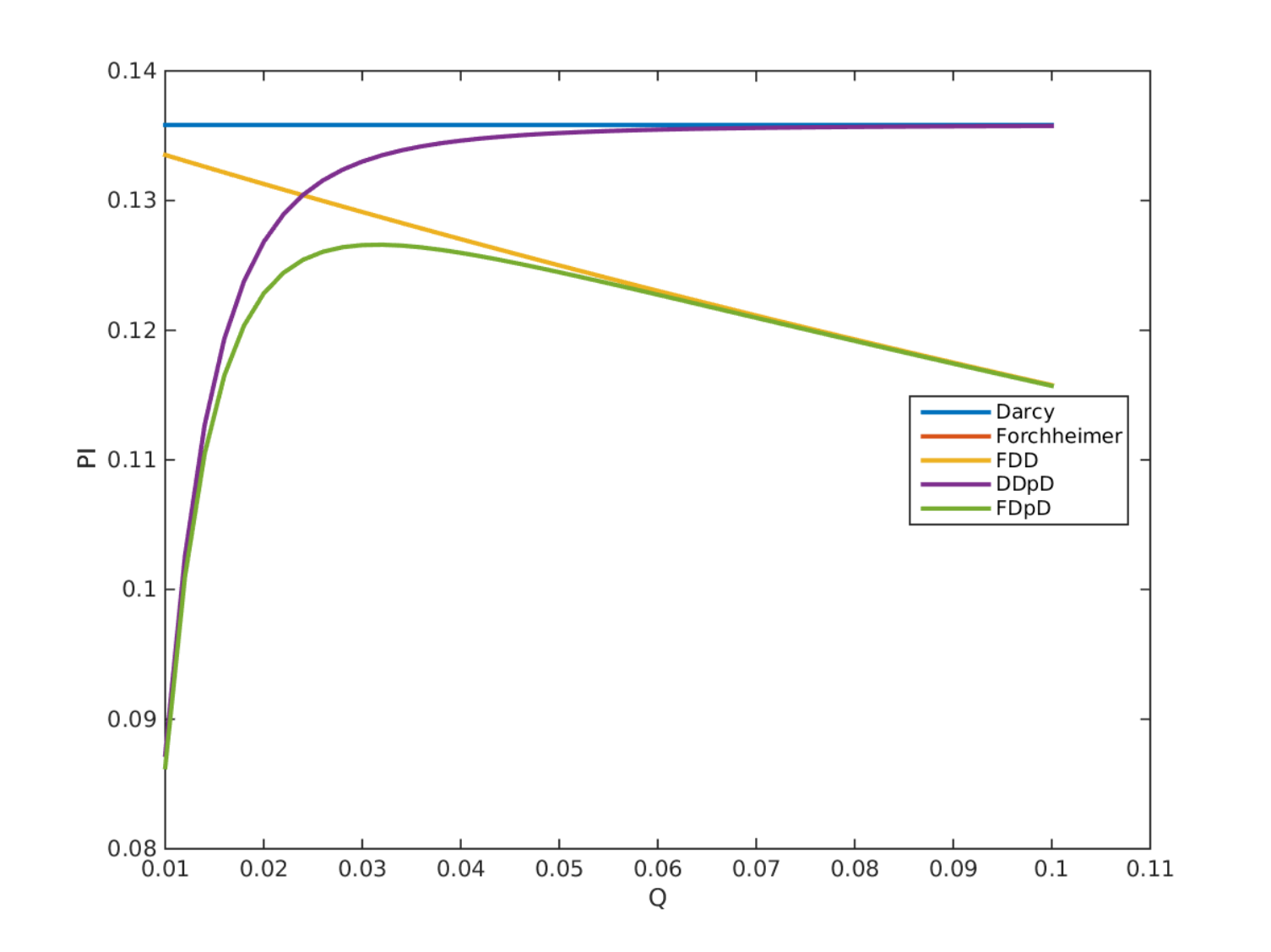
$$S_D[r_1, r_2] = \alpha \int_{r_1}^{r_2} (r_e^2 - r^2)^2 r^{-1}\, dr, \quad S_F[r_1, r_2] = \int_{r_1}^{r_2} \left[\alpha + \beta A\,(r_e^2 - r^2) r^{-1}\right] \frac{(r_e^2 - r^2)^2}{r}\, dr,$$

$$S_{pD}[r_1, r_2] = \lambda A^{-s} \int_{r_1}^{r_2} (r_e^2 - r^2)^{2-s} r^{s-1}\, dr.$$

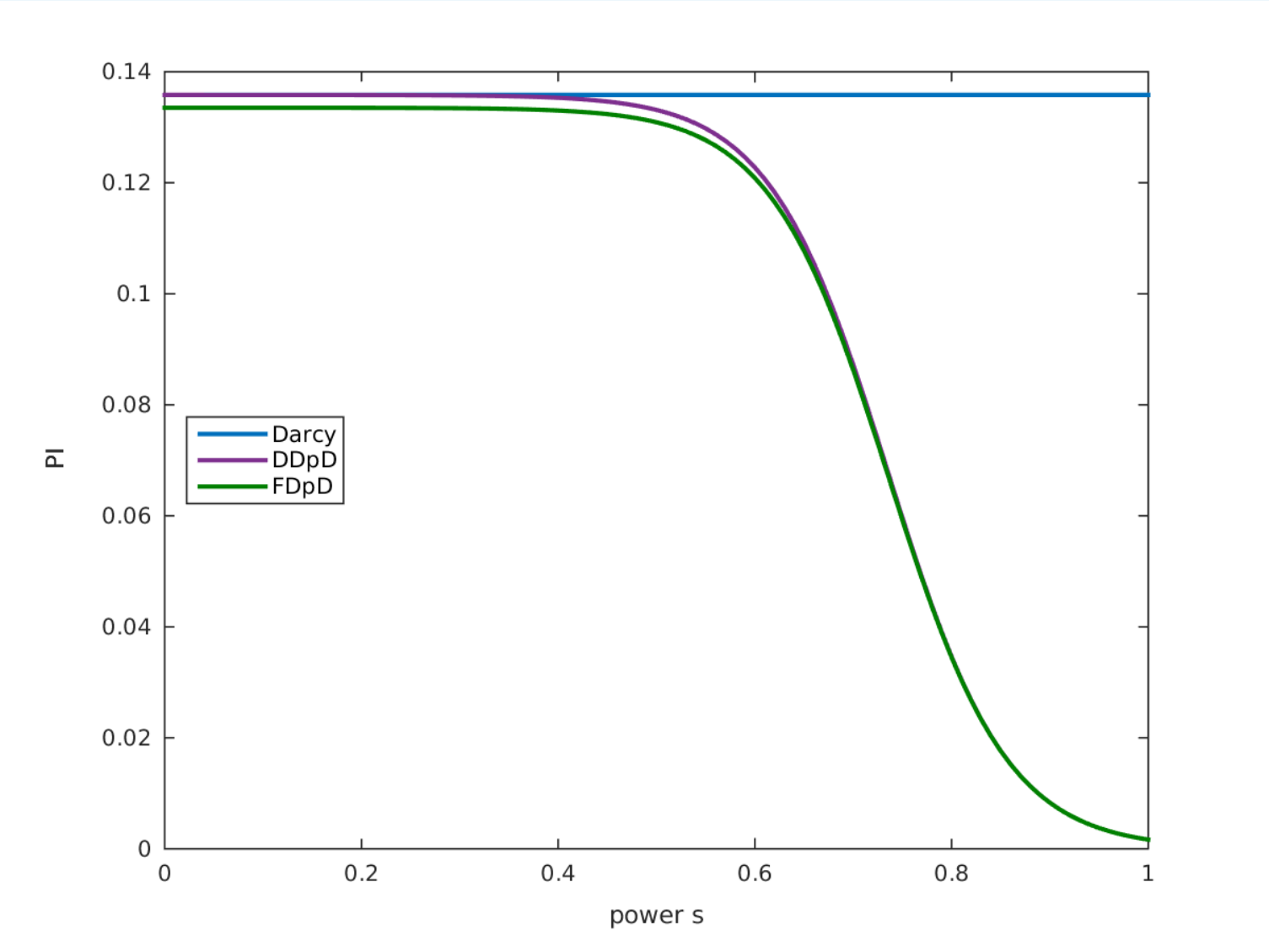
Analytical PI

- Darcy flow in all three zones: $$J_D = \frac{L}{S_D[r_w, r_e]}$$
- Forchheimer flow in all three zones: $$J_F = \frac{L}{S_F[r_w, r_e]}$$
- Forchheimer-Darcy-Darcy flow: $$\frac{L}{S_F[r_w, r_F] + S_D[r_F, r_e]}$$
- Darcy-Darcy-pre-Darcy flow: $$J_{DDpD} = \frac{L}{S_D[r_w, r_D] + S_{pD}[r_D, r_e]}$$
- Forchheimer-Darcy-pre-Darcy flow: $$\frac{L}{S_F[r_w, r_F] + S_D[r_F, r_D] + S_{pD}[r_D, r_e]}$$

PI vs. flux Q: $(s = 0.7)$



PI vs. power s:


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
PI
Darcy
DDpD
FDpD
0
0.2
0.4
0.6
0.8
1
power s

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Compressible gas: Parabolic equation for $p(x,t)$

Consider case for two terms Forchheimer polynomial: $g = \alpha u + 4\beta\rho u|u|$:

- non-linear Darcy equation:
  $u = -K(\rho|\nabla p|)\nabla p = -\frac{2}{\alpha+\sqrt{\alpha^2+4\beta\rho|\nabla p|}}\nabla p$
- the equation of state: $\rho(p) = Mp$
- the continuity equation: $\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho u)$

we get

$$\boxed{2\frac{\partial p}{\partial t} - \nabla\cdot(K(|1/2\nabla p^2|)\nabla p^2) = 2\frac{\partial p}{\partial t} - L[p] = 0}$$

For simplicity we let $M = 1$

Productivity Index for compressible gas

$$\mathrm{PI} = \frac{\text{Hydrocarbon Production}}{\text{Reservoir Pseudo Pressure} - \text{Well Pseudo Pressure}}$$

Averages over the domain and on the boundary

$$\overline{P}_U(t) = \frac{1}{|U|}\int_U p^2(x,t)\,dx \qquad \overline{P}_{\Gamma_i}(t) = \frac{1}{|\Gamma_i|}\int_{\Gamma_i} p^2(x,t)\,ds$$

$$\overline{PDD}(t) = \overline{P}_U(t) - \overline{P}_{\Gamma_i}(t)$$ *pseudo pressure drawdown*

$$Q(t) = \int_{\Gamma_i} u \cdot N\,ds$$ *total flux* through Γ_i

$$PI = J^{gas}(t) = \frac{Q(t)}{\overline{PDD}(t)}$$

<u>In contrast to slightly compressible fluid:</u> *for any initial regime $J^{gas}(t)$ is not time invariant*

however engineers use $J^{gas}(t)$ to evaluate well capacity for gas flow

Pseudo PSS Profile for compressible gas

Near-PSS profile $p_{s0}(x,t)$: PI is time-invariant and close to the original
On the boundary: $p_{s0}|_{\Gamma_i} = -\frac{Q_s}{|U|}t + B$ with total flux Q_s.

$$\frac{\partial p_{s0}}{\partial t} - L[p_{s0}] = -\frac{Q_s}{|U|}\left(\frac{1}{\sqrt{1+\frac{2W}{(B-At)^2}}} - 1\right) \text{ – equation with correction}$$

$$\frac{\partial p}{\partial t} - L[p] = 0 \qquad \text{original equation}$$

Near PSS profile/regime has a form:

$$\boxed{p_{s0}(x,t) = \sqrt{\left(\left(B - \frac{Q_s}{|U|}t\right)^2 + 2W(x)\right)}}$$

Basic profile $W(x)$ is defined on next slide, B associate to original reserves.

Pseudo PI for compressible gas

Basic profile $W(x)$

$$-\frac{Q_s}{U} = -A = \nabla \cdot (K(|\nabla W|)\nabla W), \;\; K(|\nabla W|) = \frac{2}{\alpha + \sqrt{\alpha^2 + 4\beta(|\nabla W|)}}$$

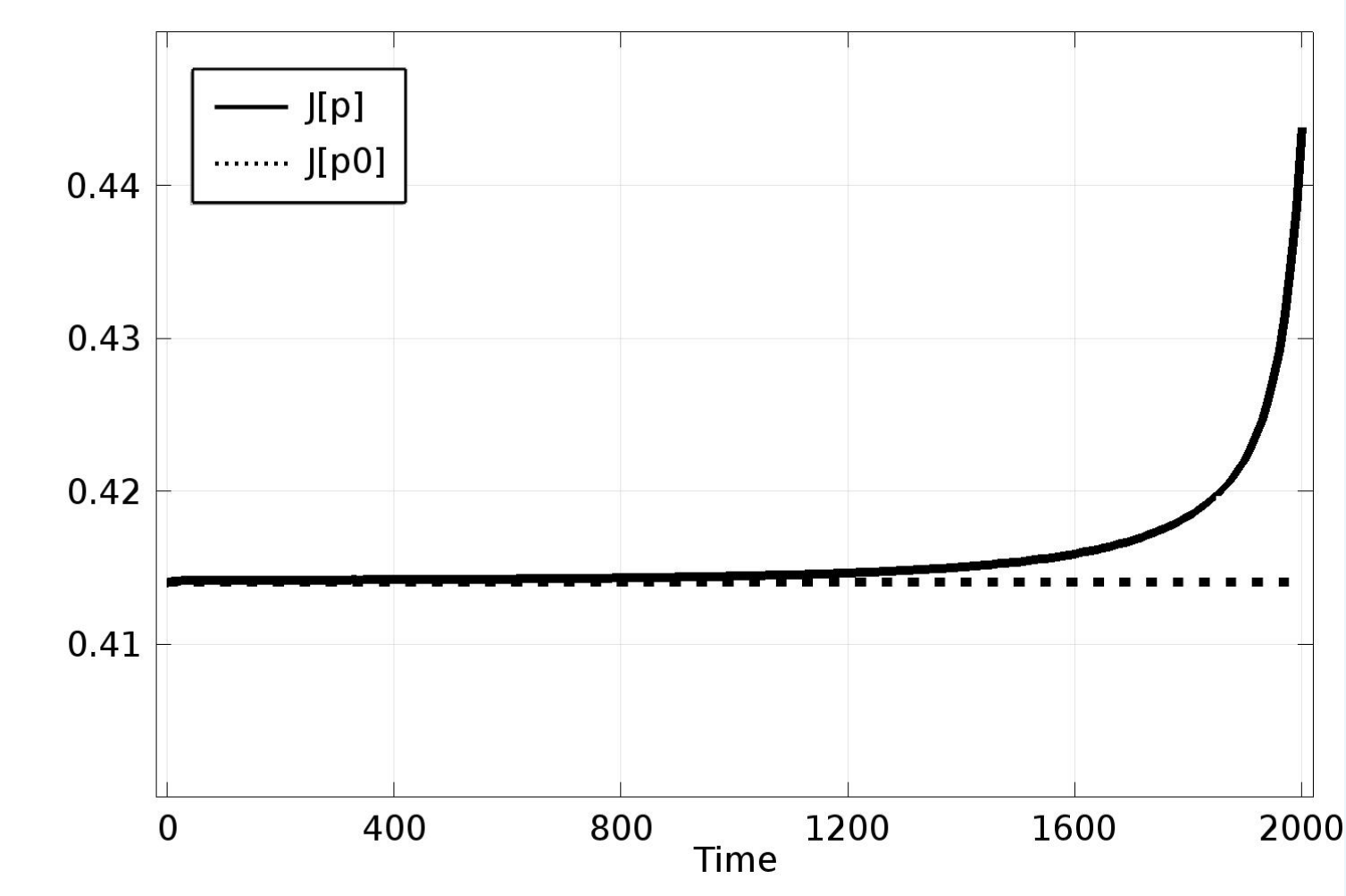
$$W(x)|_{\Gamma_i} = 0 \quad \text{and} \quad \nabla W \cdot N|_{\Gamma_e} = 0$$

Our approach is to construct profile similar original problem for which PI is constant for all time.

$$J_g^{gas} = \frac{Q_s}{\frac{1}{|U|}\int_U p_{s0}^2\,dx - \frac{1}{|\Gamma_i|}\int_{\Gamma_i} p_{s0}^2\,ds} = \frac{Q_s}{\frac{2}{|U|}\int_U W(x)\,dx}$$

Numerical proposition : Computer simulations shows that during some time $T(B, Q, U)$ depending on initial reserves value B in the reservoir U and rate of production Q_s, Pseudo PI and actual PI are almost the same.

Numerical results



$J[p](t)$ vs PSS $J[p_0]$, t approaches the critical value $T_{crit} = \frac{B}{A} = 2000$

Comparison between near PSS and original PI for compressible gas, Darcy case only

Let original problem has the same boundary and initial data as $p_{s0}(x,t)$. Namely $p(x,t) = B - At$ on the boundary for all time, and at initial moment $p(x,0) = \sqrt{B^2 + 2W(x)}$. If

$$|\nabla p(x,t)| + \Delta p(x,t) + p_t(x,t) \leq C < \infty$$

Then

$$|J_0^{gas}(B) - J^{gas}(B,t)| \leq \frac{C}{B}$$

for all time $t \leq T_{crit}/2$

Constraint on the Boundary Data

For a pressure function $p(x,t)$:

Define trace $\quad p(x,t)|_{\Gamma_i} = \psi_0(x,t) = \gamma(t) + \psi(x,t)$ (8)

Assume

$$\int_{\Gamma_i} \psi(x,t)\, ds = 0,$$

Uniqueness:

If $p_1(x,t)$ and $p_2(x,t)$ are solut-s of IBVP-I with the same $\psi(x,t)$, then

$$p_1(x,t) = p_2(x,t)$$

General Initial Pressure, and Boundary Pressure. Boundary Data Comparison

$$Q(t) \qquad \text{vs} \qquad Q_s$$

and

$$\psi(x,t) \qquad \text{vs} \qquad \varphi(x) = \varphi_0(x) - \frac{1}{|\Gamma_i|}\int_{\Gamma_i} \varphi_0(s)\, ds$$

Extensions $\Psi(x,t)$, $\Phi(x)$ defined on the whole domain U such that:

$$\Psi(x,t)|_{\Gamma_i} = \psi(x,t) \qquad \Phi(x)|_{\Gamma_i} = \varphi(x)$$

$$\Psi(x,t), \Psi_t(x,t), \Phi(x) \in W_2^1(U)$$

!!! No conditions on the average

$$\gamma(t) = \frac{1}{|\Gamma_i|} \int_{\Gamma_i} \psi_0(x,t)\, ds$$

Results are obtained under some constraints on two parameters

$$\Delta_Q(t) = Q(t) - Q_s \qquad \Delta_\Psi(x,t) = \Psi(x,t) - \Phi(x)$$

Aim:

the comparison of fully transient solution $p(x,t)$ with PSS solution $p_s(x,t)$ in terms of Diffusive Capacity $J_g(t)$

Main result:

$$J_g(t) - J_{g,PSS} \to 0 \quad \text{as} \quad t \to \infty$$

under certain assumptions on the differences in boundary data

$$\Delta_Q(t) \quad \text{and} \quad \Delta_\Psi(x,t)$$

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Lemma:

$$J_g(t) - J_{g,PSS} \to 0 \quad \iff \quad |(\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})| \to 0$$

$$\iff \int_U |\nabla(p - p_s)|^{2-a}\, dx \xrightarrow{?} 0$$

Proof:

Lemma:

$$J_g(t) - J_{g,PSS} \to 0 \quad \iff \quad |(\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})| \to 0$$

$$\iff \int_U |\nabla(p - p_s)|^{2-a}\, dx \xrightarrow{?} 0$$

Proof:

- $\|\nabla p\|_{L^{2-a}(U)} \leq C$

Lemma:

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Proof:

- $\|\nabla p\|_{L^{2-a}(U)} \leq C$
- $\|\nabla p_s\|_{L^{2-a}(U)} \leq C$

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$$\iff \int_U |\nabla(p - p_s)|^{2-a}\, dx \xrightarrow{?} 0$$

Proof:

- $\|\nabla p\|_{L^{2-a}(U)} \leq C$
- $\|\nabla p_s\|_{L^{2-a}(U)} \leq C$
- $\int_U |p_t - p_{s,t}|^2 dx \to 0 \quad \text{as} \quad t \to \infty$

Lemma:

$$J_g(t) - J_{g,PSS} \to 0 \quad \iff \quad |(\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})| \to 0$$

$$\iff \int_U |\nabla(p - p_s)|^{2-a}\, dx \xrightarrow{?} 0$$

Proof:

- $\|\nabla p\|_{L^{2-a}(U)} \leq C$
- $\|\nabla p_s\|_{L^{2-a}(U)} \leq C$
- $\int_U |p_t - p_{s,t}|^2 dx \to 0 \quad \text{as} \quad t \to \infty$
- $\int_U |p - p_s - B.C.|^2\, dx \to 0 \quad \text{as} \quad t \to \infty$

Auxiliary identity:

$$
\begin{aligned}
J_g(t) - J_{g,PSS} &= \frac{Q(t)}{\overline{p}_U - \overline{p}_{\Gamma_i}} - \frac{Q_s}{\overline{p}_{sU} - \overline{p}_{s\Gamma_i}} = \\
&= \frac{Q(t) \cdot \Delta_p(t)}{(\overline{p}_U - \overline{p}_{\Gamma_i})(\overline{p}_{sU} - \overline{p}_{s\Gamma_i})} + \Delta_Q(t)\,\frac{J_{g,PSS}}{Q_s},
\end{aligned}
$$

where

$\Delta_p(t) = (\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})$ is the difference of pressure drawdowns

Auxiliary identity:

$$
\begin{aligned}
J_g(t) - J_{g,PSS} &= \frac{Q(t)}{\overline{p}_U - \overline{p}_{\Gamma_i}} - \frac{Q_s}{\overline{p}_{sU} - \overline{p}_{s\Gamma_i}} = \\
&= \frac{Q(t) \cdot \Delta_p(t)}{(\overline{p}_U - \overline{p}_{\Gamma_i}) \underbrace{(\overline{p}_{sU} - \overline{p}_{s\Gamma_i})}_{=const.}} + \underbrace{\Delta_Q(t)}_{\text{cond.} \to 0} \frac{J_{g,PSS}}{Q_s},
\end{aligned}
$$

where

$\Delta_p(t) = (\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})$ is the difference of pressure drawdowns

Auxiliary identity:

$$
\begin{aligned}
J_g(t) - J_{g,PSS} &= \frac{Q(t)}{\overline{p}_U - \overline{p}_{\Gamma_i}} - \frac{Q_s}{\overline{p}_{sU} - \overline{p}_{s\Gamma_i}} = \\
&= \frac{Q(t) \cdot \Delta_p(t)}{(\overline{p}_U - \overline{p}_{\Gamma_i}) \underbrace{(\overline{p}_{sU} - \overline{p}_{s\Gamma_i})}_{=const.}} + \underbrace{\Delta_Q(t)}_{\text{cond.} \to 0} \frac{J_{g,PSS}}{Q_s},
\end{aligned}
$$

where

$\Delta_p(t) = (\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})$ is the difference of pressure drawdowns

$(\overline{p}_U - \overline{p}_{\Gamma_i}) \to const.$ if $\Delta_p(t) \to 0$ as $t \to \infty$

$\Delta_p(t) \to 0 \quad \text{as} \quad t \to \infty$

$$\Delta_p(t) = (\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})$$

Trace theorem + Poincaré inequality:

$$|\Delta_p(t)| \leq C \left(\int_U |\nabla(p - p_s)|^{2-a} \, dx \right)^{\frac{1}{2-a}}.$$

$\Delta_p(t) \to 0 \quad \text{as} \quad t \to \infty$

$$\Delta_p(t) = (\overline{p}_{sU} - \overline{p}_{s\Gamma_i}) - (\overline{p}_U - \overline{p}_{\Gamma_i})$$

Trace theorem + Poincaré inequality:

$$|\Delta_p(t)| \leq C \left(\int_U |\nabla(p - p_s)|^{2-a} \, dx \right)^{\frac{1}{2-a}}.$$

<u>Lemma:</u>

$$J_g(t) - J_{g,PSS} \to 0 \quad \Longleftrightarrow \quad \int_U |\nabla(p - p_s)|^{2-a} \, dx \xrightarrow{?} 0$$

Thank You!