Energy based modeling and model order reduction

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Research Center MATHEON
Mathematics for key technologies

Outline

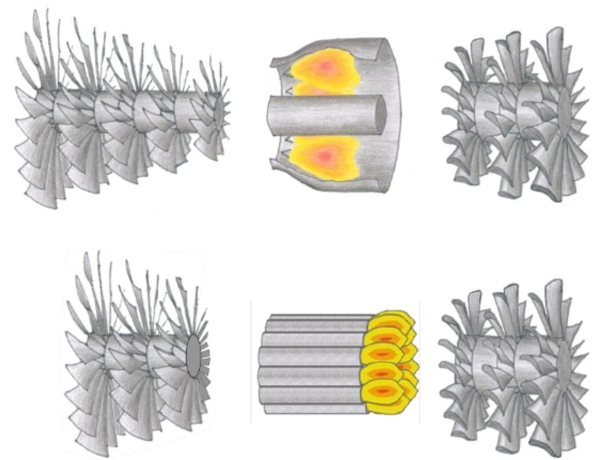
Theses

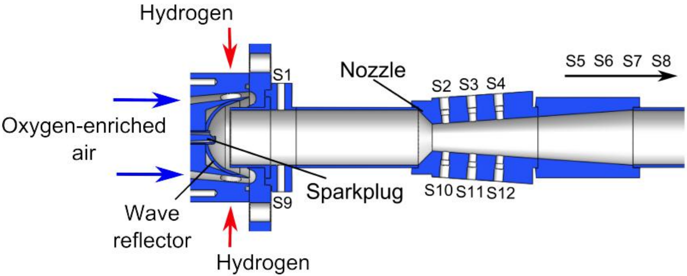
- Key technologies require **M**odeling, **S**imulation, and **O**ptimization (MSO) of complex dynamical systems.
- Most real world systems are multi-physics systems, with different accuracies and scales in components.
- Modeling today becomes exceedingly automatized, linking subsystems together.
- Modeling, analysis, numerics, control and optimization techniques should go hand in hand.
- Most real world (industrial) models are too complicated for optimization and control. Model reduction is a key issue.

Outline

A new turbine

Collaborative Research Center SFB1029 ’TurbIn’ at TU Berlin.
Significant increase of efficiency via the interactive use of instationary effects of combustion and flow in gas turbines.

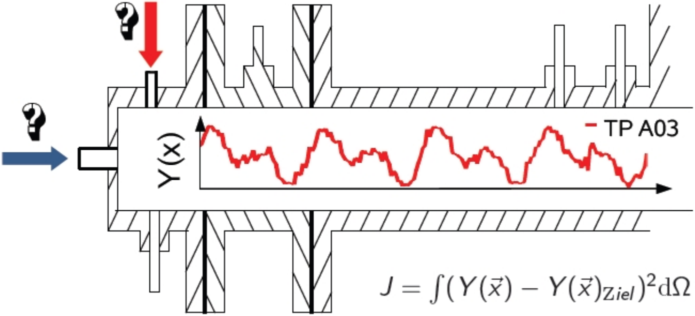




[1]

[1] Project A01, Oliver Paschereit

Optimal fuel injection



$$J = \int (Y(\vec{x}) - Y(\vec{x})_{Ziel})^2 \mathrm{d}\Omega$$

Technological Application, Tasks

Control of combustion process

- ▷ Experimental setup of combustion process.
- ▷ Modeling of turbulent reactive flow.
- ▷ Control methods for the filling and ignition of pipes.
- ▷ Control method for flows that hit the turbine blade.
- ▷ Model reduction and observer design.
- ▷ Model hierarchy and digital twin for simulation and control.

Ultimate engineering goal: 10 % more efficiency in turbine.

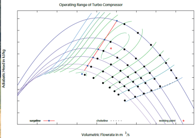
Modeling, simulation, optimization of gas networks.

- Separation of trade and transport by political regulations.
- Modeling of gas transport in large networks.
- Incorporation of weather, market, physical system, real data.
- Network planning and network operation.
- Combining discrete, stochastic, and continuous control and optimization.

Components of gas flow model

Coupled system of partial differential-algebraic equations.

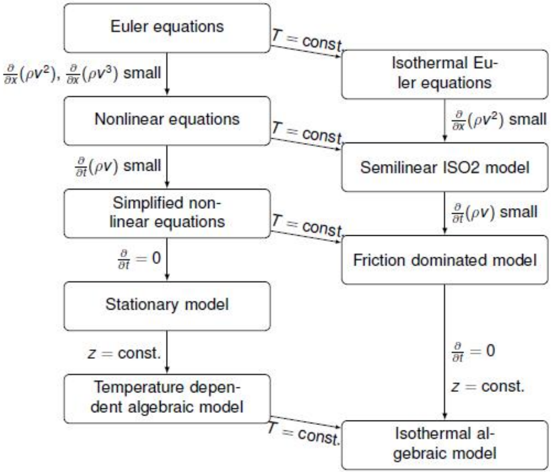
- ▷ Euler equations (with temperature) to describe flow in pipes.
- ▷ Network model, flow balance equations (Kirchoff's laws).
- ▷ Network elements: pipes, valves, controllers, heaters, compressors, coolers. Surrogate and reduced order models.



- ▷ Erratic demand and nomination of transport capacity.
- ▷ Using gas network as storage for hydrogen, methane produced from unused wind energy. **Power to gas**.

Ultimate goal: Digital twin for reliable gas flow simulation and optimization using a model hierarchy.

Model hierarchy



Model hierarchy for gas flow.
P. Domschke, O. Kolb, J. Lang (2011).

PDE constrained optimization

Different approaches.

- Simulate PDE to generate I/O surrogate model. Reduce I/O model, then optimize/control.
- First semi-discretize (in space), then reduce continuous time model, then optimize/control. (POD, Balanced truncation, DEIM, IRKA, . . .).
- Discretize (in space and time) as optimization or control problem in adaptive way (reduced basis).
- Discretize optimality conditions (forward and adjoint problem) in adaptive way (adaptive FE, FD, FV).
- Combinations of all of these.
- Apply computed control in large semi-discretized model infinite dimensional or real physical model.

- Input space $\mathcal{U}$, Output space $\mathcal{Y}$, State space $\mathcal{Z}$.
- System governed by linear or nonlinear PDE

$$
\begin{aligned}
\partial_t z &= \mathcal{A}z + \mathcal{B}u, \quad \text{in } \Omega \times [0, T], \\
z(0) &= z^0 + \text{boundary conditions}, \\
y &= \mathcal{C}z,
\end{aligned}
$$

with operators between function spaces

$$
\mathcal{B} : \mathcal{U} \to \mathcal{Z}, \; \mathcal{A} : \mathcal{Z} \to \tilde{\mathcal{Z}}, \; \mathcal{C} : \tilde{\mathcal{Z}} \to \mathcal{Y}.
$$

- System maps inputs u to outputs y.

Illustration framework

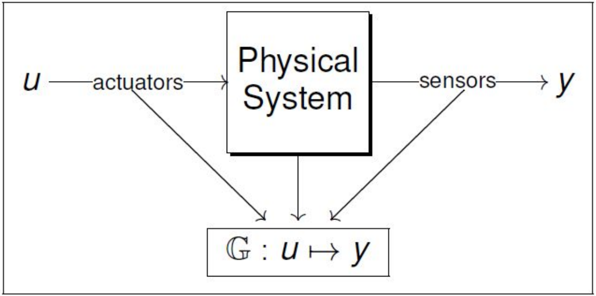


Figure: Schematic illustration of the I/O map for a physical system.

Outline

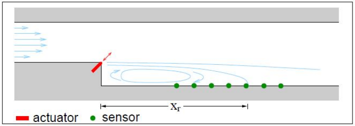
Classical and successful approach in control engineering:

- Build prototype or accurate simulator for forward problem.
- Generate I/O sequences $(u_i)_i,\ (y_i)_i$ either by measurement or by solving the PDE.
- Generate I/O map (typically in frequency domain) that interpolates the I/O sequences.
- Realize I/O map as a (small) linear finite dimensional system

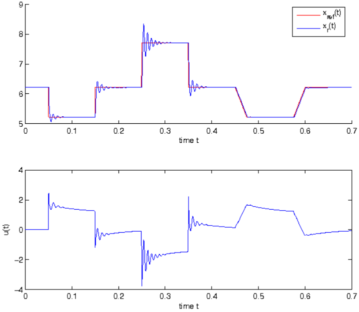
$$\dot{x} = Ax + Bu,\ \ y = Cx$$

  with matrices A, B, C.
- Build a feedback controller from the small linear model and apply it in the full physical model.

Controlled flow, backward facing step



Henning/ Kuzmin/M./Schmidt/Sokolov/Turek '07. Movement of recirculation bubble following reference curve via controller built into flow solver FEATFLOW.



Limits of classical I/O approach

- Prototypes are costly or not feasible.
- Simulators are typically for forward problem, they usually use very fine grids.
- Adaptive methods adapt for the error in the forward simulation.
- Commercial CFD codes cannot be used well.
- Fine space discretization leads to a very large system.
- For multi-physics models these models may not catch the most important part for the controller.
- Model reduction of fine model as alternative

Outline

Replace semidiscretized (in space) linear or nonlinear system

$$\begin{aligned} \frac{d}{dt} z_n(t) &= A_n z_n(t) + B_n u_m(t), \quad \text{in } \Omega \times [0, T], \\ z_n(0) &= z_n^0, \; y_p = C_n z_n, \end{aligned}$$

$z_n : [0, T] \to \mathbb{R}^n$, $u_m : [0, T] \to \mathbb{R}^m$, $y_p : [0, T] \to \mathbb{R}^p$ by ROM

$$\begin{aligned} \frac{d}{dt} z_r &= A_r z_r + B_r u_m, \quad \text{in } \Omega \times [0, T], \\ z_r(0) &= z_r^0, \; y_p = C_r z_r, \end{aligned}$$

$z_r : [0, T] \to \mathbb{R}^r$, $u_m : [0, T] \to \mathbb{R}^m$, $y_p : [0, T] \to \mathbb{R}^p$, $r << n$.

Goals

- Approximation error $\|y - y_r\|$ small, global error bounds;
- Preservation of physics: stability, passivity, conservation laws;
- Stable and efficient method for model reduction.

SVD (singular value decomposition) based methods

- ▷ Balanced truncation (linear) Antoulas, Benner, Li, Moore, Mehrmann, Penzl, Stykel, Sorensen, Varga, Wang, White, ...
- ▷ Principal orthogonal decomposition (POD), (linear/nonlinear) Banks, Benner, Hinze, King, Kunisch, Tröltzsch, Volkwein, ...
- ▷ DEIM (nonlinear) Chaturantabut, Maday, Sorensen, ...

Interpolation based methods

- ▷ IRKA (linear) Antoulas, Beattie, Gugercin, ...

Krylov methods

- ▷ Moment matching, (linear) Bai, Boley, Freund, Gallivan, Gragg, Grimme, Van Dooren, ...
- ▷ Modal truncation (linear) Bampton, Craig, Guyan, Rommes...

Reduced basis methods

- ▷ (linear/nonlinear) Haasdonk, Ohlberger, Patera, Quateroni, Rozza, ...

Proper Orthogonal Decomposition (POD)

Consider infinite dimensional

$$\begin{aligned} \frac{d}{dt}z &= \mathcal{A}z + \mathcal{B}u, \quad \text{in } \Omega \times [0, T], \\ z(0) &= z^0 + \text{boundary conditions}, \; y = \mathcal{C}z, \end{aligned}$$

or semidiscretized (in space on a fine grid) system

$$\begin{aligned} \frac{d}{dt}z_n(t) &= A_n z_n(t) + B_n u_m(t), \quad \text{in } \Omega \times [0, T], \\ z_n(0) &= z_n^0 \; y_p = C_n z_n, \end{aligned}$$

Compute snapshot matrix for well chosen input u,

$$\mathcal{X} = \left[\; z(t_1) \quad z(t_2) \quad \ldots \quad z(t_N) \; \right]$$

This has finitely or infinitely many rows.

Compute subspace V_r associated with r largest singular values of $\mathcal{X}$ by truncating small singular values σ_i, $i = r, r+1, \ldots$.
Project equations by W_r^* with $W_r^* V_r = I_r$.

$$\begin{aligned} \frac{d}{dt} z_r &= A_r z_r + B_r u_m, \quad \text{in } \Omega \times [0, T], \\ z_r(0) &= z_r^0, \; y_p = \mathcal{C}_r z_r, \end{aligned}$$

with $A_r = W_r^* \mathcal{A} V_r$, $B_r = W_r^* \mathcal{B}$, $C_r = \mathcal{C} V_r$
or
$A_r = W_r^* A_n V_r$, $B_r = W_r^* B_m$, $C_r = C_p V_r$.

- Cheap and easy to use.
- 'Works' for nonlinear systems with discrete empirical interpolation Chaturantabut, Maday, Sorensen.
- Very successful in practice.
- Can be combined with off-line computation.
- A posteriori error estimates: Kunisch/Tröltzsch/Volkwein.
- How to choose $u(t)$ for snapshots?
- Method is quite heuristic.
- Does not work well for transport dominant phenomena.
- But do we really discretize the right problem?
- Usually we do preserve physical properties, e.g. conservation laws.

Suppose we have a convolution representation of the continuous time I/O map $G : u \to y$

$$y(t) = (\mathbb{G}u)(t) = \int_0^T \mathcal{C}\mathcal{S}(t-s)\mathcal{B}u(s)\, ds$$

with kernel

$$\mathcal{K}(t-s) = \mathcal{C}\mathcal{S}(t-s)\mathcal{B}$$

where $\mathcal{S}$ is the (time continuous) solution operator for the PDE.
Idea: Discretize this I/O map, rather than the PDE.

Two step procedure

1. Approximation of input-output signals, by restricting to finite dimensional subspaces in $\mathcal{U}, \mathcal{Y}$.
2. Approximation of the dynamics/kernel

$$\mathcal{K}(t) \approx \tilde{K}$$

by approximate solution of many PDEs for the basis functions of the input space to desired tolerance.

Observations

- $\mathcal{K}(t)$ can be calculated column-wise for each input function.
- Parallelization is easy.
- No storage of state trajectories is necessary.
- Accuracy is only needed in the observations of excited states not in the states itself.
- We can easily deal with non-smooth initial transients.
- Approximate error estimation is possible, e.g. via *Dual-Weighted Residuals*
- The techniques work well for heat equations Diss. Schmidt 2007, Heiland/M./Schmidt 2011, Stokes, Oseen, linearized Navier-Stokes. Diss. Heiland 2014, Heiland/M. 2012

Lin. Navier-Stokes, Stokes

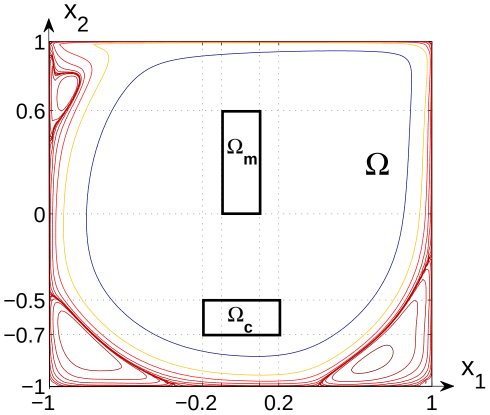
A linearization of Navier-Stokes (for Velocity V and pressure P) along a divergence-free reference velocity V_∞

$$
\begin{aligned}
V_t + (V_\infty \cdot \nabla)V + (V \cdot \nabla)V_\infty + \nabla P - \frac{1}{Re}\triangle V &= \\
(V_\infty \cdot \nabla)V_\infty + f + \mathcal{B}u, & \\
\nabla \cdot V &= 0, \\
y &= \mathcal{C}V.
\end{aligned}
$$

together with appropriate initial and boundary conditions.

Convolution operator in linear model, together with discrete input and output spaces, enables explicit construction of I/O-operator. Heiland/M. 2013, Emmrich/M. 2013, Diss. Altmann 2015.

Application to control of driven cavity flow Heiland/M. 2012



Solver IFISS by Elman/Silvester/Rammage

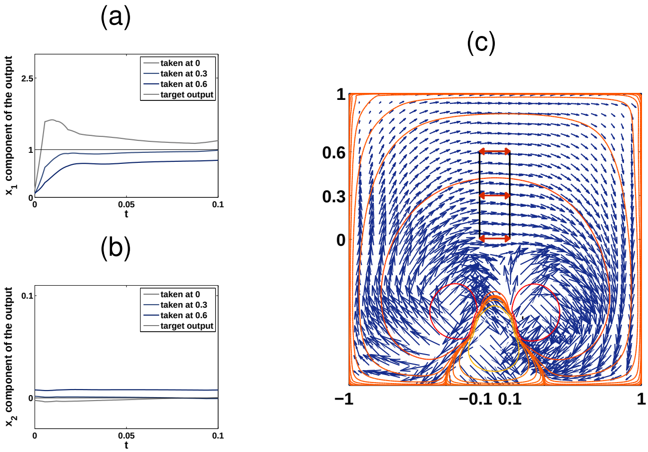


Figure: System response for input u that was computed to match an output $y^T = [1\ 0]^T$. (a) and (b) show the time evolution. Plot (c) shows the velocities and the streamlines at $t = 0.1$.

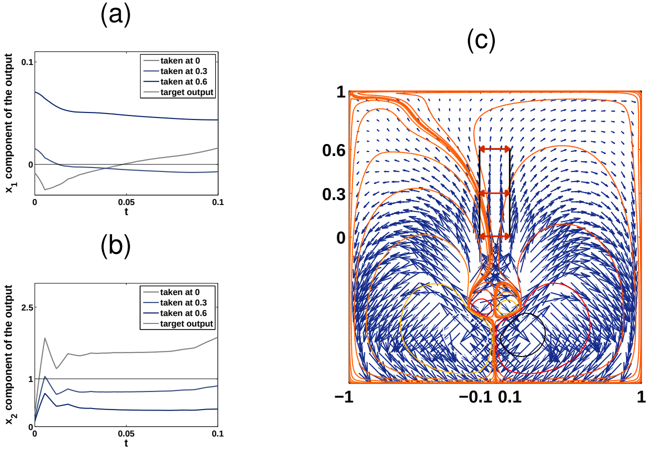


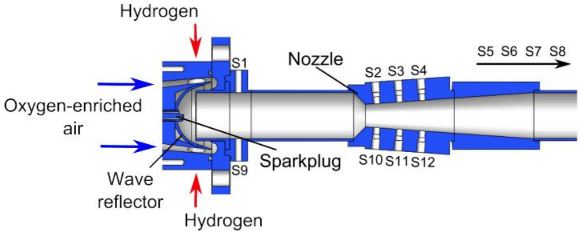
Figure: System response for input u that was computed to match an output $y^T = [0\ 1]^T$. (a) and (b) show the time evolution of the output signal. Plot (c) shows the velocities and the streamlines at $t = 0.1$.

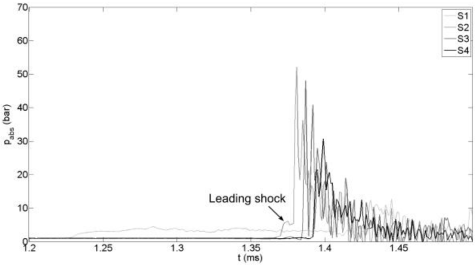
Evaluation of I/O discr. Approach

- Close to the classical control approach.
- ‘Works’ also for nonlinear systems, no theory though.
- Can be combined with off-line computation.
- Needs a representation of I/O map.
- Preservation of physical properties?
- Does not work for transport dominated problems.

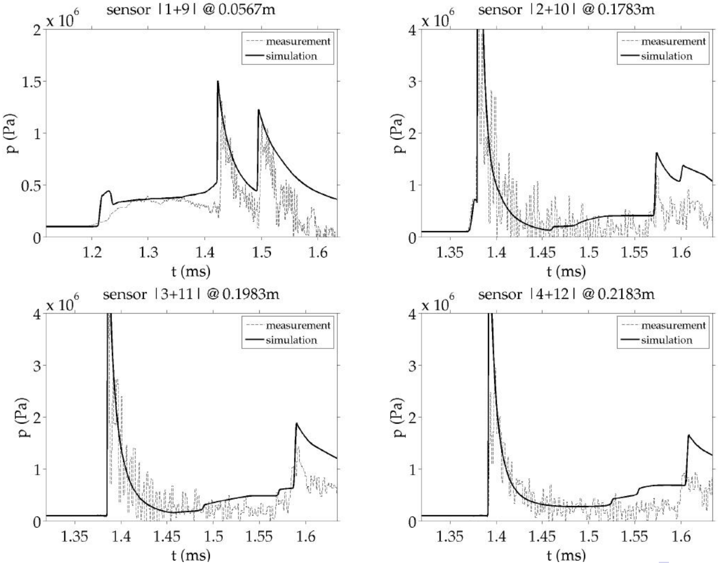
How about the new turbine?

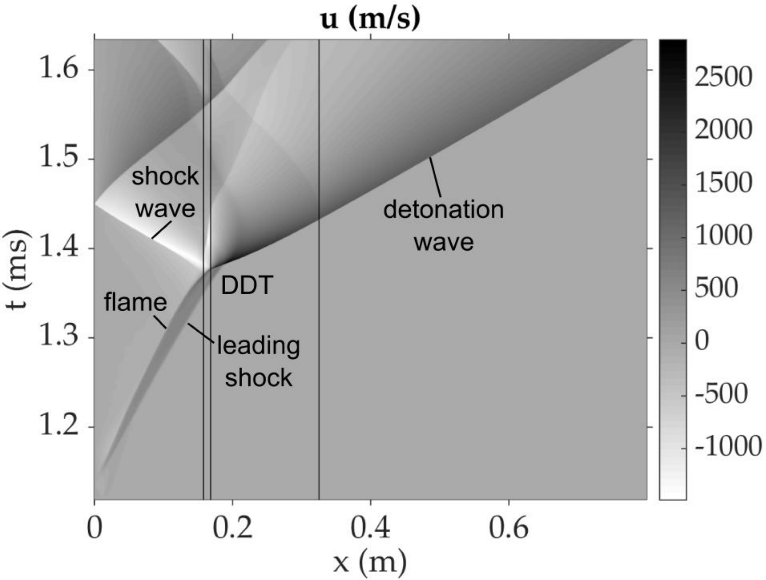
- Flow is turbulent, reactive and transport dominated.
- I/O map is highly complicated.
- All known MOR approaches fail or do not get a small model.
- We need to have reduced model that captures the transport phenomenon and the physics.


Hydrogen
Oxygen-enriched
air
Wave
reflector
Hydrogen
S1
S9
Nozzle
Sparkplug
S2 S3 S4
S10 S11 S12
S5 S6 S7 S8


S1
S2
S3
S4
Leading shock
p_{abs} (bar)
t (ms)
0
10
20
30
40
50
60
70
1.2
1.25
1.3
1.35
1.4
1.45

Reactive compressible Navier-Stokes equations.




u (m/s)
1.6
1.5
1.4
1.3
1.2
t (ms)
0
0.2
0.4
0.6
x (m)
shock
wave
flame
DDT
leading
shock
detonation
wave
2500
2000
1500
1000
500
0
-500
-1000

New approach SPOD Reiss/Schulze/Sesterhenn/M. 2015-17.

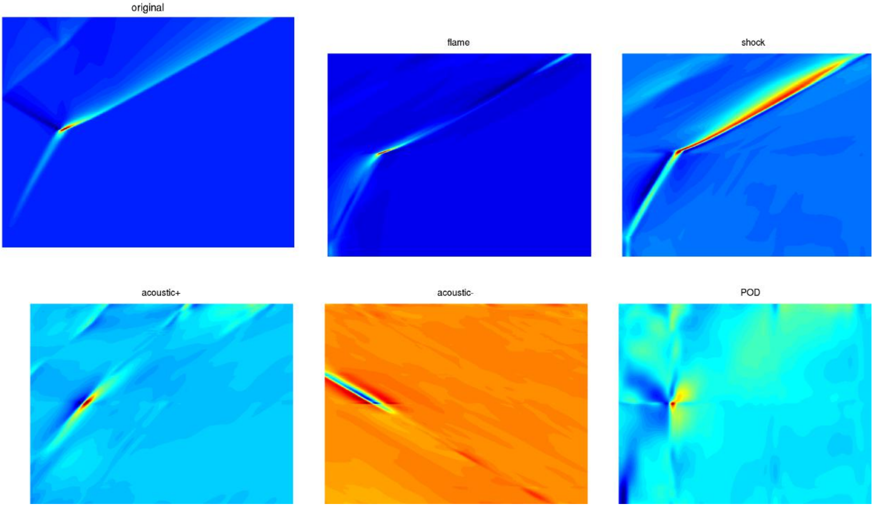
- Identify amplitudes, phases and directions of waves from SVD spectrum.
- Separate them as contributions in the transport phenomenon and do POD on the remaining components.

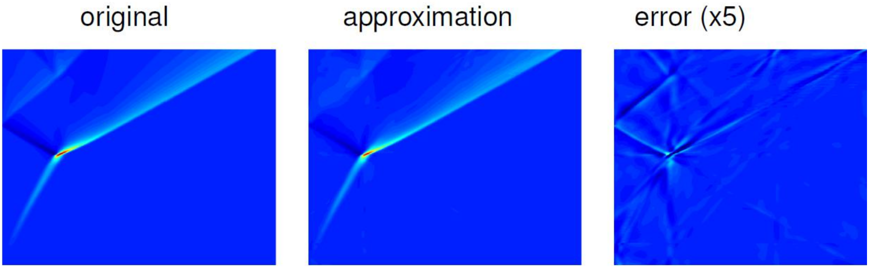
Ansatz:

$$u(x,t) = \sum_{k=1}^{N} \sum_{i} \alpha_i^k(t) \phi_i^k(x - \Delta^k(t))$$

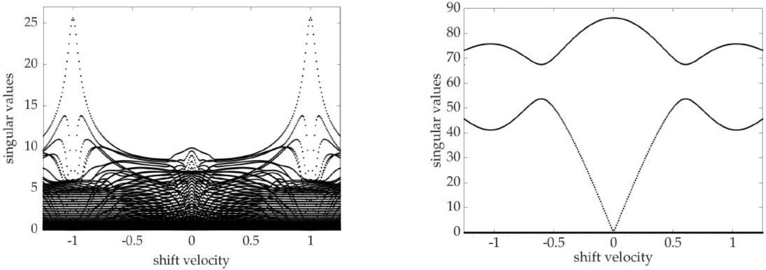
Perform Galerkin model assimilation with this ansatz.

Reduced velocity profile




original
approximation
error (x5)

Singular value spectrum to identify transport velocities.



Singular value spectrum for 1D traveling waves, 2 different velocities at ± 1 and standing wave.

- Close to the classical control approach.
- ‘Works’ for nonlinear systems.
- Can be combined with off-line computation.
- ’Works’ for transport dominated problems.
- Requires to identify transport velocities (sometimes very difficult).
- Error bounds?
- Preservation of physical properties?

Outline

Energy based modeling

Variational principles lead to energy based models: Hamiltonian systems with dissipation, inputs/outputs

- Multibody dynamics;
- Quantum mechanics;
- Electrical circuis;
- Power grids;
- Optimality systems in optimal control of ODEs/DAEs;
- Thermodynamics;
- Fluid dynamics;
- ...

Is there a common description?
$\rightarrow$ Port-Hamiltonian systems, GENERIC.

Port-Hamiltonian systems

Classical port-Hamiltonian (pH) ODE/PDE systems have the form

$$\begin{aligned} \dot{x} &= (J(x,t) - R(x,t))\,\nabla_x \mathcal{H}(x) + (B(x,t) - P(x,t))u(t), \\ y(t) &= (B(x,t) + P(x,t))^T \nabla_x \mathcal{H}(x) + (S(x,t) + N(x,t))u(t), \end{aligned}$$

- $\mathcal{H}$ is the *Hamiltonian*: it describes the distribution of internal energy among the energy storage elements;
- $J = -J^T$ describes the *energy flux* among energy storage elements within the system;
- $R = R^T \geq 0$ describes *energy dissipation/loss* in the system;
- $B \pm P$: *ports* where energy enters and exits the system;
- $S + N$, $S = S^T$, $N = -N^T$, direct *feed-through* input to output.
- In the infinite dimensional case J, R, B, P, S, N are *operators* that map into appropriate function spaces.

- Port-Hamiltonian systems generalize *Hamiltonian systems*.
- *Conservation of energy* replaced by *dissipation inequality*

$$\mathcal{H}(x(t_1)) - \mathcal{H}(x(t_0)) \leq \int_{t_0}^{t_1} y(t)^T u(t)\, dt,$$

- Port-Hamiltonian systems are closed under *power-conserving interconnection*. Models can be coupled in *modularized* way.
- Minimal pH systems are *stable and passive.*
- Port-Hamiltonian structure allows to preserve physical properties in *Galerkin projection, model reduction*.
- Physical properties encoded in *algebraic structure* of coefficients and in *geometric structure* associated with flow.
- Systems are *easily extendable* to incorporate multiphysics components: chemical reaction, thermodynamics, electrodynamics, mechanics, etc. Open/closed systems.

Discussed examples can be modeled as Port-Hamiltonian (P)DAEs.

Current work:

- Unify concept of (P)DAEs and port-Hamiltonian systems;
- Find a representation that allows automated modeling.
- Incorporate control/optimization methods.
- Develop structured discretization methods.
- Develop structured model reduction methods.
- . . .

Definition (Beattie, M., Xu, Zwart 2017)

A linear variable coefficient (P)DAE of the form

$$\begin{aligned} E\dot{x} &= [(J-R)Q - EK]\,x + (B-P)u, \\ y &= (B+P)^T Qx + (S+N)u, \end{aligned}$$

with $E, A, Q, R = R^T, K \in C^0(\mathbb{I}, \mathbb{R}^{n,n})$, $B, P \in C^0(\mathbb{I}, \mathbb{R}^{n,m})$, $S + N \in C^0(\mathbb{I}, \mathbb{R}^{m,m})$ is called *port-Hamiltonian DAE (pHDAE)* if :

i) $\mathcal{L} := Q^T E \frac{d}{dt} - Q^T JQ - Q^T EK$ is *skew-adjoint*.

ii) $Q^T E = E^T Q$ is bounded from below by a constant symmetric H_0.

iii) $W := \begin{bmatrix} Q^T RQ & Q^T P \\ P^T Q & S \end{bmatrix} \geq 0, \; t \in \mathbb{I}$.

Hamiltonian is defined as $\mathcal{H}(x) := \frac{1}{2} x^T Q^T E x : C^1(\mathbb{I}, \mathbb{R}^n) \to \mathbb{R}$.

- Analogous definition in nonlinear/ infinite dimensional case.
- Hamiltonian defines energy functional, Lypapunov function.
- Index reduction for index one and high index pHDAEs Beattie, M., Xu, Zwart 2017.
- Infinite dimensional pH systems Maschke, Ramirez, et al, Van der Schaft survey 2013, Jacob, Zwart 2012

Egger/Kugler/Liljegren/Marheineke/M. 2017 Propagation of pressure waves on the acoustic time scale in a gas network.

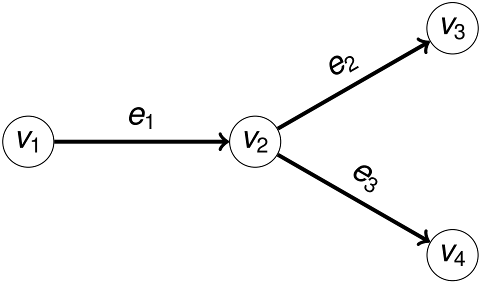


Figure: Graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertices $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$ and edges $\mathcal{E} = \{e_1, e_2, e_3\}$ defined by $e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3)$, and $e_3 = (v_2, v_4)$.

Gas transport pH-PDAE

- Model on every edge $e \in \mathcal{E}$ the conservation of mass and the balance of momentum, $z = (p, q)$.

$$\begin{aligned} a^e \partial_t p^e + \partial_z q^e &= 0, \qquad e \in \mathcal{E}, \\ b^e \partial_t q^e + \partial_z p^e + d^e q^e &= 0, \qquad e \in \mathcal{E}, \end{aligned}$$

  where p^e, q^e denote the pressure and mass flux, respectively.

- Encode in $a^e(t,z), b^e(t,z) > 0$ physical properties of fluid and pipe, in $d^e(t,z) \geq 0$ damping due to friction, and introduce interior and exterior vertices $\mathcal{V}_0$ and $\mathcal{V}_\partial = \mathcal{V} \setminus \mathcal{V}_0$.

- Model conservation of mass and momentum at $v \in \mathcal{V}_0$ by

$$\begin{aligned} \sum_{e \in \mathcal{E}(v)} n^e(v) q^e(v) &= 0 \\ p^e(v) &= p^f(v), \qquad e, f \in \mathcal{E}(v), \end{aligned}$$

  where $\mathcal{E}(v) = \{e : e = (v, \cdot) \text{ or } e = (\cdot, v)\}$ is the set of edges adjacent to v and $n^e(v) = \mp 1$ (flow direction).

- Inputs: $p^e(v) = u_v, \quad v \in \mathcal{V}_\partial,\ e \in \mathcal{E}(v)$
- Output: the mass flux in and out of the network via the exterior vertices
$$y_v = -n^e(v)q^e(v), \qquad v \in \mathcal{V}_\partial,\ e \in \mathcal{E}(v),$$
- Initial conditions: $p(0) = p_0$, $q(0) = q_0$ on $\mathcal{E}$ for pressure and mass flux.
- Hamiltonian:
$$\mathcal{H} = \frac{1}{2}\sum_{e\in\mathcal{E}} \int_e a^e|p^e|^2 + b^e|q^e|^2 dz.$$

Discontinuous Galerkin discretization

Existence of unique solution for consistent initial conditions p_0, q_0 and sufficiently smooth $(u_v)_{v\in\mathcal{V}_\partial}$, in Egger/Kugler 2016.
Mixed finite element space discretization leads to pHDAE:

$$\begin{aligned} E\dot{x} &= (J-R)Qx + Bu, \\ y &= B^T x, \\ x(0) &= x^0, \end{aligned}$$

with $Q = I$, $S, N, P = 0$,

$$E = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, J = \begin{bmatrix} 0 & -\tilde{G} & 0 \\ \tilde{G}^T & 0 & \tilde{N}^T \\ 0 & -\tilde{N} & 0 \end{bmatrix}, R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \tilde{D} & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \tilde{B}_2 \\ 0 \end{bmatrix}.$$

The Hamiltonian is given by

$$\mathcal{H}(x) = \frac{1}{2}x^T E^T Q x = \frac{1}{2}(x_1^T M_1 x_1 + x_2^T M_2 x_2).$$

Galerkin reduction for pH systems Beattie/ Gugercin 2011.
Replace

$$E\dot{x} = (J - R)\nabla_x H(x) + Bu, \; y = B^T \nabla_x H(x)$$

by reduced system

$$E_r \dot{x}_r = (J_r - R_r)\nabla_{x_r} H_r(x_r) + B_r u, \; y_r = B^T \nabla_{x_r} H_r(x_r)$$

with $x \approx V_r x_r$, $\nabla_x H(x) \approx W_r \nabla_{x_r} H_r(x_r)$, $J_r = W_r^T J W_r$, $R_r = W_r^T R W_r$ $W_r^T E V_r = E_r$, $B_r = W_r^T B$.
If V_r and W_r are appropriate orthornormal bases, then the resulting system is again pHDAE and all properties are preserved.

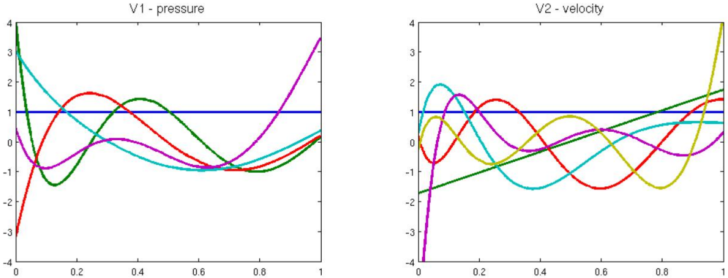
Egger/Kugler/Liljegren-Sailer/Marheineke/M. 2017.

- Algebraic compatibility conditions for full model.
- Well-posedness, conservation of mass, dissipation inequality, and exponentially stability of steady states.
- Model reduction via moment matching
- Specially structured Krylov method to satisfy algebraic compatibility conditions.
- CS Decomposition to guarantee geometric structure.
- Reduced model satisfies same conditions, no reduction of constraints.
- Efficient construction of projection spaces V_r, W_r.
- Error bounds.

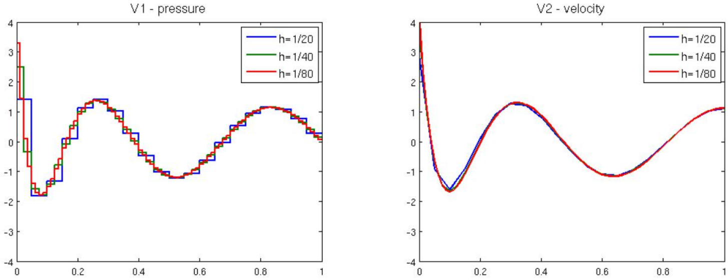
Comparison with standard method

		exact	$\mathbb{V}_i = \mathbb{W}_i^L$			$\mathbb{V}_i = \mathbb{W}_i^L + \mathbb{Z}_i$		
	L		1	3	10	1	3	10
projection	m_h	1.000	0.750	0.902	0.949	1.000	1.000	1.000
	E_h	0.500	0.375	0.451	0.475	0.500	0.500	0.500
mass constraint	m_h	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	E_h	0.500	0.667	0.554	0.527	0.500	0.500	0.500

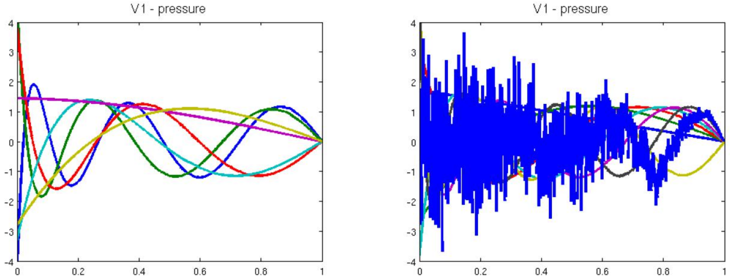
Initial values of $m_h(0)$ and $E_h(0)$ for the mass and energy for full order and reduced models obtained by projection in the energy norm with and without additional mass constraint.



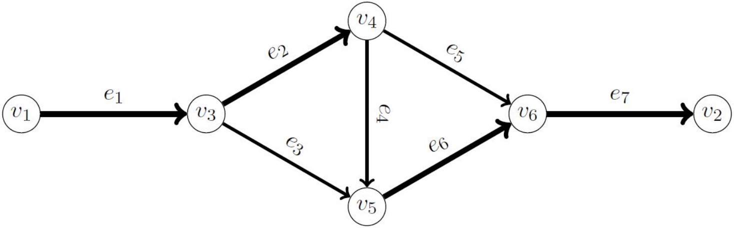
Bases for the subspaces obtained by the structure preserving Krylov iteration with $L = 4$.



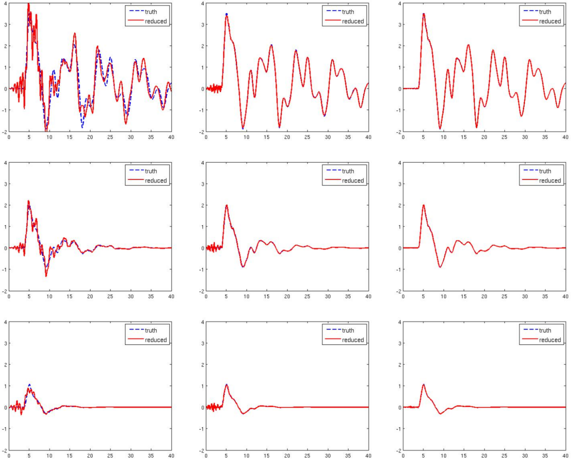
Basis functions for the pressure and velocity computed with space-discretized model on different meshes with mesh size $h = \frac{1}{20}, \frac{1}{40}$, and $\frac{1}{80}$.



With and without pressure correction via CS decomposition.


v_1
e_1
v_3
e_2
e_3
v_4
e_4
v_5
e_5
e_6
v_6
e_7
v_2

MOR via projected Krylov methods



Results for space-discretized model (blue) and reduced model (red) with dim. $2, 5, 10$ and damping param. $d = 0.1, 1, 5$ (top to bottom).

Summary

- Coupled systems from different physical domains including flow have wide applications.
- Energy based modeling via PH PDAEs a very promising approach.
- Structure is rich and allows for big improvements in analysis, numerics, control, perturbation theory.
- Space-time discretization preserving pHDAE structure.
- Model reduction preserving pHDAE structure.
- Incorporation of experimental and real time data. (Data assimilation).

The area is wide open for very interesting research

Thank you very much
for your attention
and my sponsors for their support

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- (DFG) Research center MATHEON
- Collaborative Research Centers 1029, TR154.

Details: http://www.math.tu-berlin.de/~mehrmann/

Happy birthday Zdenek and Eduard
welcome to the O60 club.

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