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AN OPTIMAL UNIFORM A PRIORI ERROR ESTIMATE FOR AN UNSTEADY SINGULARLY PERTURBED PROBLEM

MILOSLAV VLASAK AND HANS–GÖRG ROOS

Abstract. A time–dependent convection–diffusion problem is discretized by the Galerkin finite element method in space with bilinear elements on a general layer adapted mesh and in time by discontinuous Galerkin method. We present optimal error estimates. The estimates hold true for consistent stabilization too.

Key words. discontinuous Galerkin, convection–diffusion, layer adapted mesh, error estimate

We are ready to present the main result.

Theorem 1. *Let u be an exact solution of (**??**) and $U \in V_N^\tau$ be its discrete approximation given by (**??**). Then*

$$\max_{m=1,\dots,r} \sup_{I_m} \|U - u\| \le C \left(g_2(N) + \tau^{q+1} \right). \tag{1}$$

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The first author is a junior researcher of the University centre for mathematical modelling, applied analysis and computational mathematics (Math MAC).

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Charles University in Prague, Faculty of Mathematics and Physics, Sokolovská 83, 186 75 Prague, Czech Republic
E-mail: vlasak@karlin.mff.cuni.cz
URL: http://www.karlin.mff.cuni.cz/∼vlasak/

Technical University of Dresden, Institute of Numerical Mathematics, Helmholzstrasse 10, 01069 Dresden, Germany
E-mail: hans-goerg.roos@tu-dresden.de
URL: http://www.math.tu-dresden.de/∼roos/