

# OPTIMAL ERROR ESTIMATES FOR NONSTATIONARY SINGULARLY PERTURBED PROBLEMS FOR LOW DISCRETIZATION ORDERS\*

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**Abstract:** We consider unsteady 1D singularly perturbed convection–diffusion problem. We discretize such a problem by linear finite element method (FEM) on a Shishkin mesh and by discontinuous Galerkin method in time. We present optimal a priori error estimates for low order time discretizations.

**Keywords:** convection–diffusion, Shishkin mesh, time discontinuous Galerkin method.

We are ready to present the main result.

**Theorem 1** Let  $u$  be an exact solution of (??) and  $U \in V_N^\tau$  be its discrete approximation given by (??) with  $q = 0, 1$ . Then

$$\max_{m=1,\dots,r} \sup_{I_m} \|U - u\| \leq C \left( (N^{-1} \log(N))^2 + \tau^{q+1} \right). \quad (1)$$

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