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A PRIORI DIFFUSION–UNIFORM ERROR ESTIMATES FOR SINGULARLY PERTURBED PROBLEMS: MIDPOINT–DG DISCRETIZATION

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Abstract

We deal with a nonstationary semilinear singularly perturbed convection–diffusion problem. We discretize this problem by discontinuous Galerkin method in space and by midpoint rule in time. We present diffusion–uniform error estimates with sketches of proofs.

We are ready to present the main result.

Theorem 1. *Let $p > 1 + d/2$. Let $h_1, \tau_1 > 0$ are such that*

$$C_T^2(h_1^{2p+1} + \varepsilon h_1^{2p} + \tau_1^4) \leq \frac{1}{2} h_1^{2+d}. \tag{1}$$

Let τ_1 is sufficiently small to guarantee the existence and continuity of the continuated discrete solution. Then for all $h \in (0, h_1)$ and $\tau \in (0, \tau_1)$ we get

$$\sup_{t\in[0,T]} \|e(t)\|^2 \leq C_T^2(h^{2p+1} + \varepsilon h^{2p} + \tau^4), \tag{2}$$

where the constant C_T is independent of h, τ, ε.

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